

TRUNCATED PLUG NOZZLE

COMPUTER PROGRAM

DOCUMENTATION

VOLUME I

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BY

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## GENERAL COMMENTS

### 1. Non-Dimensionalization -

In this program most length coordinates are non-dimensional; the base radius (or base half-height for planar geometries) is the reference length. The method of characteristics solution does not require non-dimensionalization, but the base pressure solution does require non-dimensionalized radial coordinates. The only term which is not non-dimensional is the entropy which has the units  $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ .

### 2. References -

This documentation should be used in conjunction with: "Characteristics of Separated Flow Regions within Altitude Compensating Nozzles" by Thomas J. Mueller, Wayne P. Sule, and Charles R. Hall, Jr. Final Report UNDAS TN-029-FR-9, University of Notre Dame, Notre Dame, Indiana, January 1971. The equations referenced in this documentation correspond to the equations in this report, both in the text and in the appendices.

## OPERATING INSTRUCTIONS

The first three cards are run identification cards. These cards may contain any information desired, and will be printed out at the beginning of the run. These cards may contain any alpha-numeric characters, but cannot be punched beyond column 55. If no run identification message is desired, then blank cards must be inserted.

1. READ 107
2. READ 108
3. READ 109

The next card reads in the desired number of vertical characteristic points in the flow field. This must be an even number. NDJ is an integer number, and  $20 \leq NDJ \leq 40$ .

4. READ 117, NDJ

The next card reads in the properties of the nozzle gas at reservoir conditions. The variable G is the ratio of specific heats ( $= 1.400$  for air); RG is the gas constant ( $= 53.3 \text{ Ft-Lb}_f/\text{LB}_m - ^\circ\text{R}$  for air); TOL is the reservoir temperature in degrees Rankine; and POL is the reservoir pressure in psia. All of these numbers are real. (See Fig. 1)

5. READ 101, G, RG, TOL, POL

The following card reads in the plug base radius for axisymmetric flows or the base half-height for a planar geometry. This is a real number. Also an integer number describing the type of flow field is read in. NEPS = 0 for a planar configuration, or NEPS = 1 for an axisymmetric geometry. (See Fig. 1)

6. READ 103, RPB, NEPS

The next card reads in the ambient pressure. This real number has the units psia. (See Fig. 1)

7. READ 106, PA

The following card reads in the number of streamlines desired. There may be zero streamlines, but if this integer number is not zero, then  $2 \leq \text{NOSTRL} \leq 10$ . It is suggested that while the program is converging on a base pressure solution, zero streamlines should be used. Once a solution has been obtained then substitute the correct base pressure and then calculate streamlines.

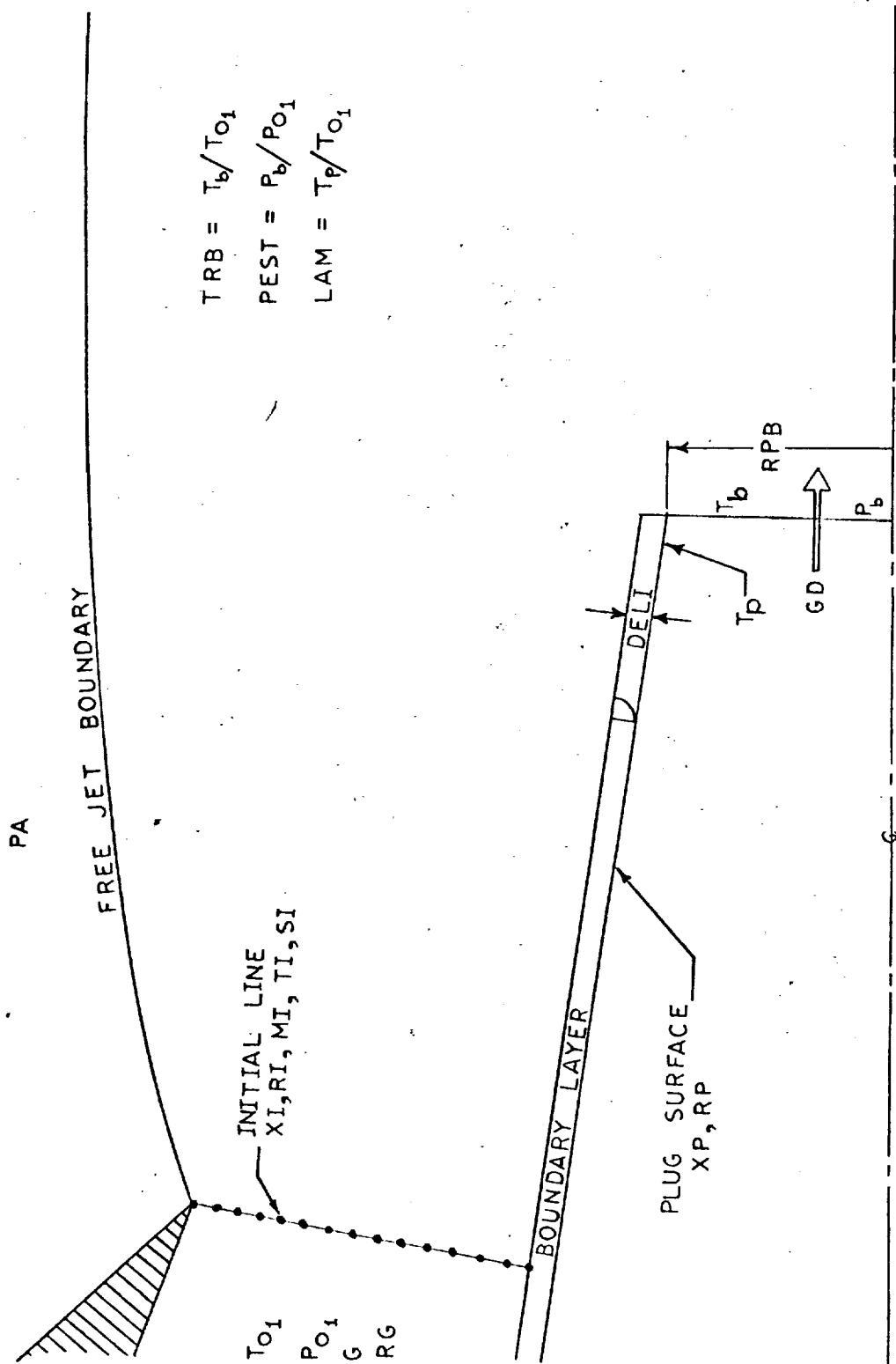


FIGURE 1

8. READ 146, NOSTRL

The next card deals with the base pressure solution. ACST4 is the accuracy requirement used in locating the j-streamline (ACST4 = 0.0001 is nominal). ACC5C6 is the accuracy requirement on the base pressure solution. (ACC5C6 = 0.001 is suggested). NASHF is the recompression coefficient, and for the present, it is suggested that NASHF = 1.00. All of these numbers are real.

9. READ 110, ACST4, ACC5C6, NASHF

The next card describes the type of near wake boundary desired in the base pressure solution. IBOUND = 1 for a conetail solution, while IBOUND = 2 for a constant pressure mixing solution.

10. READ 111, IBOUND

The next card describes the base bleed characteristics of the near wake. For no base bleed, MBLD = 0. When base bleed does occur, then MBLD = 1. MBLD is an integer number. The term GD is the actual amount of base bleed in  $Lb_m/Sec$ . (See Fig. 1) GD is a real number.

11. READ 116, MBLD, GD

The following card describes the boundary layer characteristics on the plug immediately upstream of separation. MBLDBL = 0 for no boundary layer; MBLDBL = 2 when a boundary layer is present. This is an integer number. DELI is the boundary layer thickness in inches (real number). N is a real number reading in the power law profile exponent. LAM is a real number determining the plug temperature ratio, TP/T01. (See Fig. 1)

12. READ 153, MBLDBL, DELI, N, LAM

The next card reads in the first estimate of the base pressure ratio, PB/P01, and also the base temperature ratio, TB/T01. Both of these numbers are real. (See Fig. 1)

13. READ 112, PEST, TRB

The next data card tells how many plug contour points are to be used in (NOPPTS), and also the number of initial points which are to be read in (NOIPTS).  $3 \leq \text{NOPPTS} < 50$ ;  $3 \leq \text{NOIPTS} \leq 99$ . Also, the type of initial line is determined by the variable IOPTR. IOPTR = 1 for a non-characteristic initial line, or IOPTR = 2 for a right-running characteristic read-in. All three numbers are integer.

14. READ 102, NOPPTS, NOIPTS, IOPTR

The next set of cards read in the plug coordinates, XP and RP. Each

card contains one set of points. A total of NOPPTS cards will be read in. XP and RP are real numbers, and are in inches. (See Fig. 1)

15. READ 104, XP(I), RP(I)

The last set of data cards read into the program contains the variables along the initial profile. XI and RI are the coordinates of the point in inches. MI is the Mach number at that point; TID is the streamline angle in degrees; SI is the entropy at that point in  $\text{Ft} - \text{Lb}_f/\text{Lb}_m - ^\circ\text{R}$ . A total of NOIPTS cards will be read in. (See Fig. 1)

16. READ 105, XI(J), RI(J), MI, TID, SI(J)

## COMMON BLOCKS

The following is an alphabetized list of the COMMON blocks which are used in this program. The terms in the COMMON blocks are arranged in their order of appearance in the block and are not in alphabetical order. The names of the individual items may vary from program to program, but the order of the items remains the same; the most common name of each element is used.

/ACCBLK/ ACC5C6 - accuracy requirement for the base pressure solution

/AMB/ PA - ambient pressure ( $\text{lb/in}^2$ )  
MAMB - Mach number corresponding to ambient conditions

/BBBLK/ H - non-dimensional bleed number

/BBLK/ B - result of Equation 2

/BLBK/ LAM - plug temperature ratio,  $T_p/T_{01}$   
EX - boundary layer velocity profile exponent  
CSQ - square of the Crocco number adjacent to the boundary layer just upstream of separation  
DELI - boundary layer thickness (inches)  
GDBL - equivalent bleed due to the boundary layer ( $\text{lb}_m/\text{sec}$ )  
DL2ST - boundary layer momentum thickness (inches)  
GDRAT - ratio of base bleed to the sum of base bleed and equivalent base bleed due to the boundary layer  
N - denominator of the boundary layer velocity profile exponent

/BLDM/ MBLD - integer denoting whether base bleed is present  
GD - base bleed rate ( $\text{lb}_m/\text{sec}$ )

/BLK3A/ IBOUND - integer denoting which type of base pressure solution is desired  
MESHPM - factor used in setting a minimum number of discrete turns from the plug surface to the near wake

/CH1BLK/ IOPTR - integer denoting the type of initial line which is to be read into the program

/CNRANG/ T2 - streamline angle (radians) at the last point on the plug surface

/CNTR/ DI - non-dimensional distance along the starting line  
XI - non-dimensional axial coordinate at each point along the starting line  
RI - non-dimensional radial coordinate at each point along the starting line

NUI	- Prandtl-Meyer angle (radians) at each point along the starting line
TI	- streamline angle (radians) at each point along the starting line
SI	- entropy at each point along the starting line ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )
/CORNER/	
X2	- non-dimensional axial location of the plug base, OR the non-dimensional axial location of recompression
R2	- non-dimensional plug base radius, OR the wake radius ratio
I	- subscript
NTC	- number of discrete turns in going from the plug surface to the near wake
DTC	- incremental change in streamline angle (radians) in going from the plug surface to the near wake
/CSBLK/	
CS	- square of the Crocco number just upstream of recompression
/DATBLK/	
M1A	- Mach number on the plug surface just upstream of separation
ACST4	- accuracy requirement used in locating the j-streamline
/D3BLK/	
PHID3	- velocity ratio on the d-streamline at recompression
/ETABLK/	
ETAJ3	- non-dimensional coordinate locating the j-streamline at recompression
PHIJ3	- velocity ratio along the j-streamline at recompression
ETAM2D	- non-dimensional shift between the viscous and inviscid coordinate systems just downstream of separation
NASHF	- recompression coefficient
M3AM2A	- ratio of the Mach number just upstream of recompression to that just after separation
M2A	- Mach number just downstream of separation
/F4BLK/	
CR	- ratio of the Crocco number just upstream of recompression to that just after separation
CJ2	- value of the J2 integral (Equation 15)
CI2	- value of the I2 integral (Equation 13)
FP	- pressure function
GP	- geometric parameter (right hand side of Equation 1)
EGP	- geometric parameter (left hand side of Equation 1)
ETAM3D	- non-dimensional shift between the viscous and inviscid coordinate systems just upstream of recompression
/GAS/	
G	- ratio of specific heats
RG	- gas constant ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )



TOL - chamber stagnation temperature ( $^{\circ}\text{R}$ )  
 POL - chamber stagnation pressure ( $\text{lb}/\text{in}^2$ )

/PARAM/ X - non-dimensional array of axial coordinates at each point in the characteristics matrix  
 R - non-dimensional array of radial coordinates at each point in the characteristics matrix  
 NU - array of Prandtl-Meyer angles (radians) at each point in the characteristics matrix  
 T - array of streamline angles (radians) at each point in the characteristics matrix  
 S - entropy at each point in the characteristics matrix ( $\text{ft}\cdot\text{lb}_f/\text{lb}_m\cdot^{\circ}\text{R}$ )

/PBBLK/ PBPO - base pressure ratio,  $P_b/P_{O1}$

/PLCBLK/ ILOC - integer denoting which portion of the flow field is being calculated (See Fig. IV-1)  
 LOC - integer denoting which portion of the flow field is being calculated (See Fig. I-3)

/POLIP/ POLP - stagnation pressure downstream of the "lip shock" ( $\text{lb}/\text{in}^2$ )  
 LSHK - integer denoting whether a "lip shock" is present

/PTNOS/ NOPPTS - number of plug coordinate points  
 NOIPTs - number of initial line points

/SIGBLK/ S3A - jet spread parameter  
 C6 - Crocco number on the d-streamline at recompression (Equation 9)

/SIZE/ NDI - maximum number of I-subscripts in the characteristics matrix  
 NDJ - maximum number of J-subscripts in the characteristics matrix

/SKLIP/ IRECP - integer denoting whether recompression has been reached

/SOLBLK/ SOL3A - ratio of the Mach number just upstream of recompression to that just after separation, OR the streamline angle (radians) just upstream of recompression  
 RSRB - wake radius ratio

/STRBLK/ NOSTRL - number of streamlines  
 NOSPTS - maximum number of points on each streamline

/STRL/ XSTR - array of axial coordinates (non-dimensional) at each streamline point  
 RSTR - non-dimensional array of radial coordinates at each streamline point

MSTR - array of Mach numbers at each streamline point  
 TSTR - array of streamline angles (radians) at each streamline point  
  
 /STRSRT/ NSTRT - array of starting points for each streamline  
  
 /THETBK/ THET12 - change in streamline angle (radians) between the plug surface and the near wake  
  
 /TPN/ XP - array of non-dimensional axial coordinates at each point on the plug surface  
 RP - array of non-dimensional radial coordinates at each point on the plug surface  
 TP - array of streamline angles (radians) at each point on the plug surface  
 XPF - non-dimensional axial location of the plug base  
 RPB - plug base radius (inches)  
  
 /TRBBLK/ TRB - base temperature ratio,  $T_b/T_{o1}$

OUTPUT:

1. The first items which are printed are the contents of the three run identification cards which were read into the program.
2. The next line gives the values of the ratio of specific heats,  $G$ , the chamber stagnation temperature in  $^{\circ}\text{R}$ ,  $T01$ , the chamber stagnation pressure in  $\text{lb/in}^2$ ,  $P01$ , and the ambient pressure in  $\text{lb/in}^2$ ,  $PA$ .
3. The nozzle plug coordinates are printed next. The dimensions of the axial coordinate,  $XP$ , and the radial coordinate,  $RP$ , are inches. A total of  $NOPPTS$  sets of coordinates will be printed.
4. On the next page, the initial starting line conditions are printed. This includes the point number, the axial and radial coordinates in inches, the Prandtl-Meyer angle in degrees, the streamline angle in degrees, and the entropy in  $\text{ft-lb}_f/\text{lb}_m-^{\circ}\text{R}$  at each point. A total of  $NDJ$  points are printed. These results are interpolated from the initial line data which was read into the program.
5. The initial estimate of the base pressure ratio,  $P_b/P_1$  is printed (from  $FBASE6$ ).
6. The next printed line gives the Mach number just upstream of separation,  $M1A$ , and the wake radius ratio,  $RSRB$ .
7. The following line prints the value of the Mach number just after separation, and the change in streamline angle (radians) in going from the plug surface to the near wake.
8. The next line prints the value of the jet spread parameter, and also the geometric parameter.
9. The next line prints out the value of the Crocco number,  $C2A$ , just after separation, the corresponding Mach number,  $M2A$ , the Crocco number just upstream of recompression,  $C3A$ , its corresponding Mach number,  $M3A$ , and the streamline angle of the near wake (radians) immediately after separation. (Items 6-9 all were printed by the function subprogram  $FBASE6$ .)
10. The next sets of numbers involve obtaining a solution locating the  $j$ -streamline. These results are printed by the subroutine  $SOTE2B$ , and the set of points correspond to  $X$  and  $SOL2$  in this subroutine. The term  $X$  corresponds to the non-dimensional coordinate, and  $SOL2$  is the difference between the geometric parameters which are calculated by two different methods.

11. The last set of two points in that column of numbers correspond to the value of the non-dimensional coordinate which locates the j-streamline, ETAJ3, and the velocity ratio on that streamline, PHIJ3. The function subprogram FBASE6 prints these results.
12. The last line of points in this section refers to the convergence of the base pressure solution. The first item is the difference between the Crocco numbers on the d-streamline at recompression, FBASE6. The second and third terms are the actual values of these Crocco numbers, C5 and C6, which are calculated by two different methods. When the term FBASE6 is less than the accuracy requirement ACC5C6, a base pressure solution has been reached.
13. Items 5-12 are repeated until a base pressure solution has been obtained (FBASE6 < ACC5C6).
14. The next section will print out the statement: IN CALC X3 < X1 or R3 < 0 several times. The number of times that this statement is printed is an indication of the number of corrections which are made on the location of the recompression shock. The numbers following this statement correspond to X3, X1 and R3.
15. The next section prints out the variables along the recompression shock. This includes the point number, the axial and radial coordinates in inches, the upstream and downstream Mach numbers, the upstream and downstream streamline angles in degrees, the shock wave angle in degrees, and the upstream and downstream entropies in  $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$  at each point.
16. The next section prints out the variables along each streamline. These include the axial and radial coordinates in inches, the Mach number, and the streamline angle in degrees at each point along the streamline. A total of NOSTRL streamlines will be printed.
17. The final section describes the final results of the solution. The first part summarizes the input which was used. The first line gives the ratio of specific heats, G, and the gas constant in  $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ .
18. The next line gives the chamber stagnation temperature in  $^\circ\text{R}$  and in  $^\circ\text{F}$ .
19. The following line presents the chamber stagnation pressure, PO1, in  $\text{lb}/\text{in}^2$  and in  $\text{lb}/\text{ft}^2$ . Also the ambient pressure is given in  $\text{lb}/\text{in}^2$  and  $\text{lb}/\text{ft}^2$ .

20. The next line prints out the base temperature ratio,  $T_b/T_{O1}$ .
21. The following line prints out the amount of base bleed in  $lb_m/sec$ .
22. The next line prints out the non-dimensional bleed number corresponding to base bleed. If no base bleed occurs, this statement is suppressed.
23. The next section prints out the results of the initial boundary layer. If no boundary layer is present, then a statement to this effect is printed and outputs 24-27 are omitted.
24. The next line prints the boundary layer thickness in inches and its momentum thickness in inches.
25. The next line gives the boundary layer profile exponent and the plug temperature ratio,  $T_p/T_{O1}$ .
26. The following line prints the amount of equivalent base bleed due to the boundary layer in  $lb_m/sec$ .
27. The next line prints the equivalent non-dimensional bleed number due to the boundary layer.
28. The next line tells the type of near wake boundary, i.e., a contail or a constant pressure boundary.
29. The following line gives the value of the recompression coefficient which was used.
30. The next section gives three base pressure ratios. These include  $P_b/P_{O1}$ ,  $P_b/P_{at}$ , and  $P_b/P_1$ .
31. The Mach number upstream of separation,  $M_{1A}$ , the Mach number just after separation,  $M_{2A}$ , and the Mach number just upstream of recompression,  $M_{3A}$ , are printed on the next line.
32. The next line prints the change in streamline angle in degrees from the plug surface to the near wake, and also the streamline angle in degrees upstream of recompression.
33. The next line prints the values of the jet spread parameter, and the wake radius ratio.
34. The following line prints the values of the Crocco number just after separation,  $C_{2A}$ , just upstream of recompression,  $C_{3A}$ , and along the d-streamline at recompression,  $C_{3D}$ .

35. The next line locates the j-streamline at recompression. PHIJ3 is the velocity ratio on this streamline, and ETAJ3 is its non-dimensional coordinate. If base bleed or a boundary layer is present, then the velocity ratio on the d-streamline, PHID3, and its non-dimensional coordinate are also printed.
36. For axisymmetric configurations, the next line prints the geometric parameter, and the non-dimensional coordinate shifts just after separation, ETAM2D, and at recompression, ETAM3D.

I. SUBROUTINE TPNZZL

## I. SUBROUTINE TPNZZL

Subroutine TPNZZL calculates the streamline angle at each given point on the plug contour, performs the first region of method of characteristics solution if the initial conditions are read in along a right-running characteristic from the shroud tip, and also sets up the characteristics mesh required to proceed further downstream.

### COMMON BLOCKS

COMMON blocks AMB, BLK3A, CH1BLK, CNRANG, CNTR, CORNER, DATBLK, GAS, PARAM, PLCBLK, POLIP, PTNOS, SIZE, SKLIP, and TPN are used.

### CALLING PROGRAM

The subroutine TPNZZL is called by the main program, MAIN.

TPNZZL calls subroutines and functions BASE6, CALC, FLOW, LIPSHK, PMANGL, SETUP, and TAB.

### FORTRAN SYSTEM ROUTINES

Built-in FORTRAN functions ATAN, MOD, and SQRT are used.

### CALLING SEQUENCE

The calling sequence is

CALL TPNZZL (NEPS, PEST)

$$\text{NEPS} = \begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$$

PEST is the initial estimate of the base pressure ratio.



ACST4 - accuracy requirement used in locating the j-streamline

AMBNU - the Prandtl-Meyer angle (radians) to which the flow must accelerate when it exits from the nozzle

A1 - the coefficient of the  $x^2$  term in the quadratic fit of the plug boundary points

A2 - the coefficient of the X term in the quadratic fit of the plug boundary points

D - the distance along the starting line (array)

DD - the incremental distance along the starting line

DEGNU - the Prandtl-Meyer angle in degrees (used to output results)

DENOM - the denominator of Cramer's Rule calculations for the quadratic fit of the plug points

DI - array of the distances along the starting line

DIST - incremental distance between points on the starting line

DLFT - remaining distance along the starting line (= 3% of total distance along the starting line)

DLNU - change in Prandtl-Meyer angle (radians) from the last point on the contour boundary to the ambient conditions

DR - incremental distance along the starting line

DREM - remaining distance along the starting line (= 97% of total distance along the starting line)

DRP - incremental distance along the starting line

DTC - the incremental change in streamline angle in going from the plug to the near wake (See Fig. IV-3)

DTH - incremental change in streamline angle (radians) in going from the contour to ambient conditions

DTOT - total distance along the starting line

EK - real number used to set up algorithm for the calculation of characteristics

EKJ - real number used to set the J-subscript in the calculation of characteristics

FTJ     - is used to calculate incremental changes in the starting line  
 G       - ratio of specific heats  
 I       - subscript  
 IBOUND - integer denoting what type of base pressure solution is desired  
 ILOC    - an integer which tells what portion of the flow field is being  
           calculated (See Fig. IV-1)  
 IOPTR   - integer describing the type of starting line  
 IRECP   - integer indicating whether recompression has been encountered  
 ISTRT   - starting value of the I-subscript  
 J       - subscript  
 JREM     - remaining J-subscripts  
 JST      - starting value of the J-subscript  
 JSTRT   - starting value of the J-subscript  
 K       - subscript  
 LOC      - an integer which tells what portion of the flow field is being  
           calculated (See Fig. I-3)  
 LSHK     - integer which tells whether a "lip shock" is present  
 MAMB     - Mach number corresponding to NUAMB  
 MESHPM   - minimum number of turns into which the expansion from the plug  
           to the near wake may occur  
 MI       - integer used to determine whether I is odd or even  
 MJST     - integer used to determine whether JST is odd or even  
 MK       - integer used to determine whether K is odd or even  
 M1A      - Mach number on the plug surface just before separation  
 NDF      - final value of the I-subscript  
 NDI      - number of I-subscripted points in the characteristic matrix  
 NDI-1   - NDI-1

NDIM2 - NDI-2  
 NDJ - number of J-subscripted points in the characteristic matrix  
 NDJML - NDJ-1  
 NDJM2 - NDJ-2  
 NDJP1 - NDJ+1  
 NDJP2 - NDJ+2  
 NDJ2 - one half of NDJ  
 NJ2P1 - NDJ2+1  
 NOIPTS - number of initial data line points  
 NOPPTS - number of plug points  
 NPPTM1 - NOPPTS-1  
 NTC - the number of discrete turns in going from the plug surface to the near wake (See Fig. IV-3)  
 NU - array of Prandtl-Meyer turn angles (radians) at each characteristic point  
 NUI - value of the Prandtl-Meyer angle (radians) at each point along the initial line  
 PA - ambient pressure ( $\text{lb/in}^2$ )  
 PBPO - base pressure ratio,  $P_o/P_{o1}$   
 POLP - stagnation pressure ( $\text{lb/in}^2$ ) downstream of the "lip shock"  
 POPA - ambient pressure ratio,  $P_{o1}/P_{at}$   
 POL - chamber stagnation pressure ( $\text{lb/in}^2$ )  
 PR - static pressure ratio on the plug surface just before separation  
 PRNU - Prandtl-Meyer angle (radians) used to set up a characteristic read-in of initial data line  
 PRTH - streamline angle (radians) used to set up a characteristic read-in of initial data line  
 R - the non-dimensional array of the radius at each characteristic point

REMJ - remaining J-subscripts  
 RG - gas constant  
 RI - value of the non-dimensional radius at each point along the initial line  
 RIR - radius in inches of characteristic point (used for output)  
 RP - array of non-dimensional plug radii coordinates  
 RPB - plug base radius (inches)  
 R2 - non-dimensional base radius  
 S - array of entropy at each characteristic point ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )  
 SI - array of entropy at each initial point ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )  
 SL - slope of the plug at each plug point  
 SOD - sum of the distances along the starting line  
 T - streamline angle (radians) at each characteristic point  
 TI - streamline angle (radians) at each initial point  
 TID - streamline angle (degrees) along the initial line (used for output)  
 T01 - stagnation temperature ( $^\circ\text{R}$ )  
 TP - streamline angle (radians) at each point on the plug  
 T2 - streamline angle at the plug base  
 X - non-dimensional array of axial coordinates at each characteristic point  
 XI - non-dimensional array of axial coordinates at each point on the initial data line  
 XIR - axial coordinates (inches) along the initial data line (used to output results)  
 XP - non-dimensional array of axial coordinates at each plug point  
 XPF - non-dimensional axial location of the plug base  
 X2 - non-dimensional axial location of the plug base

## SOLUTION METHOD

Set up position integers

1.  $LOC = 0$

2.  $ILOC = 0$

Define calculational integers

3.  $NDIM2 = NDI - 2$

4.  $NDIM1 = NDI - 1$

5.  $NDJM2 = NDJ - 2$

6.  $NDJM1 = NDJ - 1$

7.  $NDJP1 = NDJ + 1$

8.  $NDJP2 = NDJ + 2$

9.  $NPPTM1 = NOPPTS - 1$

Fit a second order curve  $(RP = (A1)(XP)^2 + (A2)(XP) + A3)$  through each three points and take derivative  $(RP_x = 2(A1)(XP) + A2)$  to determine the slope. A1 and A2 are calculated by Cramer's Rule.

10. DO 17 I = 2, NPPTM1

Determine the value of the denominator matrix at each point.

11.  $DENOM = XP(I-1)^2(XP(I) - XP(I+1)) - XP(I-1)(XP(I)^2 - XP(I+1)^2) + XP(I)^2XP(I+1) - XP(I+1)^2XP(I)$

Calculate the values of the coefficients A1 and A2 at each point.

12.  $A1 = (RP(I-1)(XP(I) - XP(I+1)) - XP(I-1)(RP(I) - RP(I+1)) + RP(I)XP(I+1) - RP(I+1)XP(I))/DENOM$

13.  $A2 = (XP(I-1)^2(RP(I) - RP(I+1)) - RP(I-1)(XP(I)^2 - XP(I+1)^2) + XP(I)^2RP(I+1) - XP(I+1)^2RP(I))/DENOM$

Calculate the slope and the streamline angle at that point.

14.  $SL = 2(A1)(XP(I)) + A2$

15.  $TP(I) = ATAN(SL)$

Determine the streamline angle at the end points.

16. If  $I = 2$ ,  $TP(1) = ATAN(2(A1)XP(1) + A2)$

17. If  $I = NPTM1$ ,  $TP(NOPPTS) = ATAN(2(A1)XP(NOPPTS) + A2)$

The streamline angle at the plug base is defined as another variable.

18.  $T2 = TP(NOPPTS)$

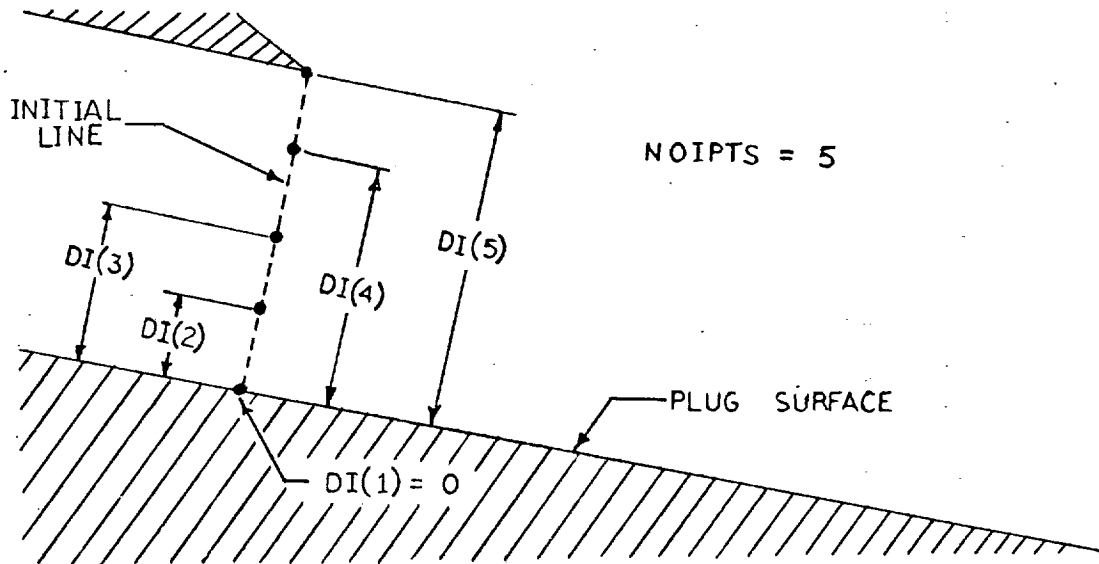


FIGURE I-1

Set up the initial data line for the characteristics solution.  
First determine the linear distances between the given points.  
(See Fig. I-1)

19.  $SOD = 0$

20.  $DI(J) = 0$

21. DO 24 J = 2, NOIPTS

22.  $DIST = \sqrt{(XI(J) - XI(J-1))^2 + (RI(J) - RI(J-1))^2}$

23.  $SOD = SOD + DIST$

24.  $DI(J) = SOD$

Redefine the total linear distance.

25.  $DTOT = DI(NOIPTS)$

Determine the ambient conditions

26.  $POPA = POL/PA$

27.  $MAMB = \sqrt{\left(\frac{2}{G+1}\right) \left(POPA^{\frac{G-1}{G}} - 1\right)}$

A non-characteristic starting line is desired (IOPTR = 1).

28. If IOPTR = 1, GO TO 90

Perform the method of characteristics solution to a non-characteristic starting line. First, determine the starting points.

29.  $ISTRT = NDJ$

30.  $JSTRT = NDJ/2$

Determine the value of the Prandtl-Meyer angle which corresponds to the given ambient conditions.

31.  $AMBNU = PMANGL(MAMB, G)$

Calculate the incremental change in streamline angle in expanding about the shroud tip to the ambient conditions.

32.  $DLNU = AMBNU - NUI(NOPTS)$

33.  $DTH = DLNU/NDJM1$

A two-dimensional expansion occurs at this point. The physical location of this point remains the same, but the Prandtl-Meyer (P-M) angle varies as does the streamline angle. The original values of the P-M angle and the streamline angle must be reduced by DTH such that an algorithm may be used.

34.  $PRNU = NUI(NOPTS) - DTH$

35.  $PRTH = TI(NOPTS) - DTH$

The term EKJ is used such that the index J is incremented once while the index I is incremented twice. The properties of number truncation from real to integer are used to accomplish this.

36.  $EKJ = JSTRT + 0.60$

37. DO 45 I = ISTRT,NDIM1

38.  $EKJ = EKJ + 0.50$

39.  $J = EKJ$

40.  $X(I,J) = XI(NOIPS)$

41.  $R(I,J) = RI(NOIPS)$

42.  $NU(I,J) = PRNU + DTH$

43.  $T(I,J) = PRTH + DTH$

Reset the streamline and P-M angles

44.  $PRNU = NU(I,J)$

45.  $PRTH = T(I,J)$

Now set the actual variation of the initial conditions from the shroud exit to the plug surface. DD is the spacing between points.

46.  $DD = DI(NOIPS)/NDJML$

Start from the top surface

47.  $D = DI(NOIPS)$

48.  $EKJ = JSTRT + 1.10$

Determine the position of the flow field variables at points along the starting line using the function routine TAB with a linear interpolation.

49. DO 55 I = NDJPL,NDIM1

50.  $EKJ = EKJ - 0.50$

51.  $X(I,J) = TAB(D,DI,XI,NOIPS,1)$

52.  $R(I,J) = TAB(D,DI,RI,NOIPS,1)$

53.  $NU(I,J) = TAB(D,DI,NUI,NOIPS,1)$

54.  $T(I,J) = TAB(D,DI,TI,NOIPS,1)$

55.  $S(I,J) = TAB(D,DI,SI,NOIPS,1)$

Print out the results of this starting line. Skip page.

56. PRINT 101

Print title

57. PRINT 102



Print heading

58. PRINT 103

Set up subscripts

59. EKJ = 1.10

60. DO 68 K = 1,NDJ

61. I = NDJ - K

62. EKJ = EKJ + 0.50

63. J = EKJ

Dimensionalize position coordinates.

64. XIR = X(I,J) (RPB)

65. RIR = R(I,J) (RPB)

Change angles from radians to degrees.

66. TID = (57.2957795)TI(I,J)

67. DEGNU = (57.2957795)NU(I,J)

Print out results

68. PRINT K,XIR, RIR,DEGNU, TID,S(I,J)

Skip a page

69. PRINT 101

The method of characteristics solution begins to calculate the flow field. The solution ends on a non-characteristic line at a constant value of the subscript I. (See Fig. I-2)

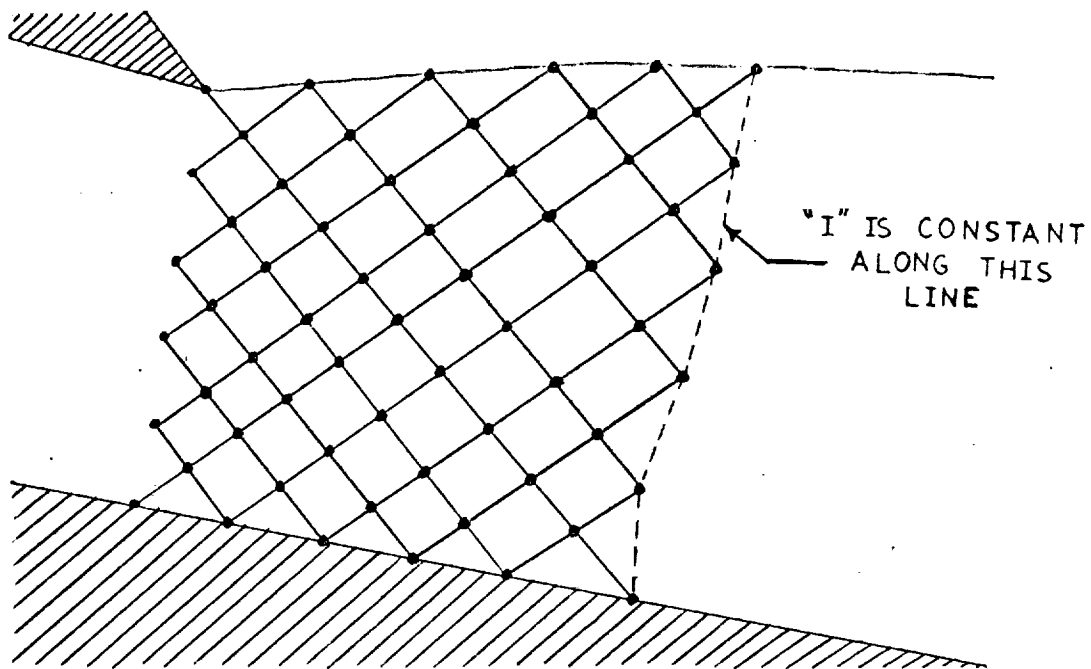


FIGURE 1 - 2

Set up subscript counter.

70.  $EK = JSTRT + 1.10$

71. DO 86 K = NDJP1,NDIM2

72.  $EK = EK - 0.50$

73.  $JST = EK$

Determine if JST is odd or even.

74.  $MJST = MOD(JST, 2)$

Determine if K is odd or even.

75.  $MK = MOD(K, 2)$

Determine J subscript for K being an odd number.

76.  $EKJ = JST + 0.10$

Determine EKJ for K being even.

77. If  $MK = 0$ ,  $EKJ = JST - 0.40$

78. DO 86 I = K,NDIM2

Determine if I is odd or even.

79. MI = MOD(I,2)

80. EKJ = EKJ + 0.50

81. J = EKJ

See if I is even

82. If MI = 0, GO TO 85

I is an odd number. Calculate point.

83. CALL CALC

84. GO TO 86

I is an even number. Calculate point.

85. CALL CALC

86. CONTINUE

Set index of last point

87. NDF = NDIM1

Reset increments of final non-characteristic line as input to next set of characteristics calculations.

88. CALL SETUP

89. GO TO 124

This section sets up the increments when a non-characteristic read-in is used. Half of the points are placed in the upper 3% of the initial line to obtain accuracy near the expansion.

90. NDJ2 = NDJ/2

Calculate number of remaining points.

91. JREM = NDJ - NDJ2

Calculate remaining distance.

92. DREM = (0.97)DTOT

Change remaining number of points to a real number.

93.  $REM J = JREM$

Calculate increment in larger region.

94.  $DR = DREM/REM J$

Calculate remaining distance.

95.  $DLFT = (0.03)DTOT$

Determine number of points left and the increment in this region.

96.  $FTJ = NDJ2/FTJ$

97.  $DRP = DLFT/FTJ$

Locate first point on plug surface.

98.  $D = 0.00$

99.  $X(1,1) = XI(1)$

100.  $R(1,1) = RI(1)$

101.  $NU(1,1) = NUI(1)$

102.  $T(1,1) = TI(1)$

103.  $S(1,1) = SI(1)$

Calculate an additional calculational number.

104.  $NJ2P1 = NDJ2 + 1$

Set up the initial line.

105. DO 113 J = 2,NDJ

Determine increments

106.  $DD = DR$

107. If  $J > NJ2P1, DD = DRP$

Determine distance along line.

108.  $D = D + DD$

Calculate position and variables of the starting line using TAB with a linear interpolation.

109.  $X(1,J) = \text{TAB}(D,DI,XI,\text{NOIPTS},1)$   
110.  $R(1,J) = \text{TAB}(D,DI,RI,\text{NOIPTS},1)$   
111.  $NU(1,J) = \text{TAB}(D,DI,NUI,\text{NOIPTS},1)$   
112.  $T(1,J) = \text{TAB}(D,DI,TI,\text{NOIPTS},1)$   
113.  $S(1,J) = \text{TAB}(D,DI,SI,\text{NOIPTS},1)$

Print out dimensionalized results.

Skip page.

114. PRINT 101

Print title

115. PRINT 102

Print headings

116. PRINT 103

117. DO 121 J = 1,NDJ

Dimensionalize position coordinates.

118.  $XIR = X(1,J) \text{ (RPB)}$

119.  $RIR = R(1,J) \text{ (RPB)}$

Get angles from radians to degrees.

120.  $\text{DEGNU} = (57.2957795) (NU(1,J))$

121.  $\text{TID} = (57.2957795) (T(1,J))$

Print out results.

122. PRINT J, XIR,RIR,DEGNU,TID,S(1,J)

Skip page

123. PRINT 101

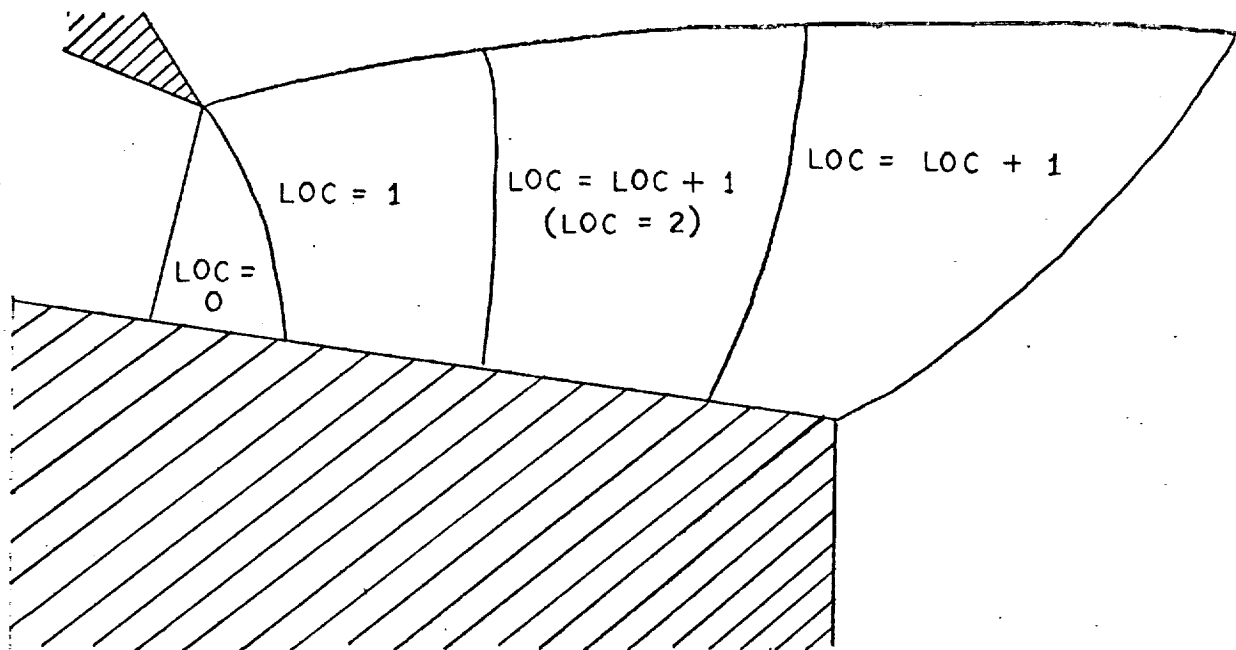


FIGURE I- 3

Update position integer (See Fig. I-3)

124.  $LOC = LOC + 1$

Calculate remainder of method of characteristics solution up to plug base.

125. CALL FLOW

Determine static pressure ratio at plug tip upstream of separation.

126.  $PR = (1 + \frac{G-1}{2} M_1^2)^{\frac{G}{G-1}}$

Determine the initial estimate of the base pressure ratio,  $P_b/P_1$

127.  $PEST = (PEST)(PR)$

Begin base pressure solution.

128. CALL BASE6

A compressive turn will result if  $P_b > P_1$ . The subroutine LIPSHK calculates the shock generated at this point.

129. If LSHK = 1 and IRECP = 0, CALL LIPSHK

Return

## II. SUBROUTINE BLAYER

## II. SUBROUTINE BLAYER

Subroutine BLAYER calculates the momentum thickness and the equivalent mass bleed due to a finite boundary layer upstream of separation.

### COMMON BLOCKS

COMMON blocks BLBK, GAS, and TPN are used.

### TPNZZL SUBROUTINES

Subroutine FLOW calls BLAYER if an initial boundary layer is present.

BLAYER calls subroutines and functions FMACH, PMTURN, PRFLE, and SIMR.

### FORTRAN SYSTEM ROUTINES

Built-in FORTRAN functions COS and SQRT are used.

### CALLING SEQUENCE

The calling sequence is:

CALL BLAYER (NEPS,NUI,TIN,DLST2,\$)

$$NEPS = \begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$$

NUI is the Prandtl-Meyer angle of the flow adjacent to the boundary layer just upstream of separation.

TIN is the streamline angle (in radians) of the flow adjacent to the boundary layer just upstream of separation

DLST2 is the momentum thickness of the boundary layer.



AREA - plug base area ( $\text{in}^2$ )  
 CSQ - square of the Crocco number adjoined to the boundary layer just upstream of separation  
 DELI - boundary layer thickness (inches)  
 DELTAI - boundary layer thickness (inches)  
 DL2ST - boundary layer momentum thickness (inches)  
 EX - velocity profile exponent  
 FMCH - Mach number function  
 G - ratio of specific heats  
 GDBL - equivalent mass bleed due to the boundary layer  
 GDRAT - ratio of base bleed to equivalent boundary layer mass flow plus base bleed  
 G1 - an equation containing specific heats  
 G2 - an equation containing specific heats  
 LAM - plug temperature ratio,  $T_p/T_{o1}$   
 MA - Mach number adjacent to the boundary layer  
 MAS - square of MA  
 N - velocity profile exponent denominator  
 PA - ambient pressure ( $\text{lb/in}^2$ )  
 PI - 3.14159265 (a constant)  
 PO1 - chamber stagnation pressure ( $\text{lb/in}^2$ )  
 RG - gas constant  
 RP - non-dimensional array of plug radial coordinates  
 RPB - plug base radius (inches)  
 TO1 - chamber stagnation pressure ( $^{\circ}\text{R}$ )  
 TP - array a plug surface angles (radians)

XP - non-dimensional array of plug axial coordinates

XPF - non-dimensional axial location of the plug base

## SOLUTION METHOD

Redefine the boundary layer thickness.

1.  $\Delta TAI = \Delta LI$

Define the constant,  $\pi$

2.  $PI = 3.14159265$

Calculate the power law profile exponent.

3.  $EX = 1/N$

Determine the Mach number on the plug surface just before separation.

4.  $CALL PMTURN$

5.  $MAS = MA^2$

Calculate the square of the Crocco number at this point.

6.  $CSQ = MAS / (MAS + (2 / (G + 1)))$

Calculate the momentum thickness using function routine

PRFLE describing the boundary layer momentum thickness equation,  
and SIMR, a Simpson's Rule integration routine.

7.  $DL2ST = SIMR (PRFLE, 0.00, \Delta TAI, 30)$

8.  $DLST2 = DL2ST$

Calculate the average boundary layer radius.

9.  $RP = RPB + (0.50) (\Delta TAI) (\cos(TIN))$

Calculate the boundary layer area for axisymmetric or planar configurations.

10.  $AREA = (2) (PI) (RP)$

11. If  $NEPS = 0$ ,  $AREA = 2.0$

Determine the mass flow function at the given pressure, temperature,  
and Mach number conditions at separation. This is done through  
the function routine FMACH

12.  $FMCH = FMACH(MA)$

Calculate the boundary layer momentum thickness. First calculate two ratios of G.

13.  $G1 = G/(G-1)$

14.  $G2 = (G-1)/2$

Calculate the static pressure just before separation.

15.  $PA = POI(1 + G2(MAS))^{G1}$

Calculate the effective mass bleed rate due to the boundary layer. (Equation 22)

16.  $GDBL = (PA)(DL2ST)(AREA)(FMACH)/SQRT(T01)$

Return to the statement number given as the 5th argument in the calling program.

17. RETURN 5

If difficulty was encountered in the Subroutine PMTURN, the program makes a normal return

18. RETURN

### III. SUBROUTINE BASE6

### III. SUBROUTINE BASE6

Subroutine BASE6 directs the calculation of the base pressure and flow field downstream of the plug base. This subroutine does very few of the calculations; its primary function is to call the calculating subroutines.

#### COMMON BLOCKS

COMMON blocks ACCBLK, BLDM, DATBLK, GAS, PBBLK, PLCBLK, and SKLIP are used.

#### TPNZZL SUBROUTINES

Subroutine TPNZZL calls BASE6.

BASE6 uses subroutines and function subprograms FBASE6, FLOW, HYPER, LINEAR, and LIPSHK

#### FORTRAN SYSTEM ROUTINES

No built-in FORTRAN functions are used.

#### CALLING SEQUENCE

CALL BASE6 (PBPO,\$,NEPS,PEST)

PBPO is the base pressure ratio,  $P_b/P_{O1}$

NEPS =  $\begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$

PEST is the original estimate of the base pressure ratio,  $P_b/P_{O1}$

ACC5C6 - accuracy requirement on the base pressure solution

ACST4 - accuracy requirement in locating the j-streamline

G - ratio of specific heats

GD - base bleed rate ( $\text{lb}_m/\text{sec}$ )

G1 - equation using the ratio of specific heats

ILOC - an integer which tells what portion of the flow field is being calculated (See Fig. IV-1)

IRECP - an integer denoting whether recompression has been encountered

LL4 - an integer used as a filler in a subroutine call

LOC - an integer which tells what portion of the flow field is being calculated (See Fig. I-3)

MBLD - an integer denoting whether base bleed and/or a boundary layer is present

MLA - Mach number on the plug surface just before separation

PBPO1 - base pressure ratio,  $P_b/P_{O1}$

PBP1 - base pressure ratio,  $P_b/P_1$

POP1 - static pressure ratio,  $P_{O1}/P_1$

PO1 - chamber stagnation pressure ( $\text{lb}/\text{in}^2$ )

PlPO - static pressure ratio,  $P_1/P_{O1}$

RG - gas constant

T - test for convergence in Subroutine LINEAR

TOL - stagnation temperature ( $^{\circ}\text{R}$ )

## SOLUTION METHOD

Converge on a base pressure solution. This is done using the function routine FBASE6 as an argument. The solution curve for base bleed differs from that of no base bleed. Therefore, two separate function routines are used.

1. If MBLD = 0, CALL LINEAR
2. If MBLD = 0, CALL HYPER
3. GO TO 5

Return to calling program if problems developed in FBASE6.

4. RETURN 2

Calculate final base pressure ratio. First calculate ratio of G.

5.  $G1 = G/(G-1)$

Determine the static pressure ratio upstream of separation.

6.  $POP1 = (1 + ((G-1)/2)MIA^2)^{G1}$
7.  $PlPO = 1/POP1$

Calculate the base pressure ratio.

8.  $PBPO = (PBPl)(PlPO)$
9.  $PBP01 = PBPO$

Redefine the location integer.

10. ILOC = 4

Once the base pressure has converged, the remainder of the flow field is calculated. If both the lip shock and the recompression shock are present in the same characteristic mesh, then IRECP = 1

11. If IRECP = 0, CALL FLOW
12. If IRECP = 1, CALL LIPSHK
13. RETURN.



#### IV. SUBROUTINE FLOW

#### IV. SUBROUTINE FLOW

Subroutine FLOW directs the actual calculations of the method of characteristics solution for different regions of the nozzle flow field. The solution extends to recompression and upward along a left-running characteristic from the recompression point.

##### COMMON BLOCKS

COMMON blocks AMB, BLBK, BLDM, BLK3A, CH1BLK, CNRANG, CNTR, CORNER, DATBLK, GAS,PARAM, PLCBLK, PTNOS, SIZE, SOLBLK, THETBK, and TPN are used.

##### TPNZZL SUBROUTINES

Subroutine FLOW is called by subroutine TPNZZL for the flow field up to the plug base; by the function subprogram FBASE6 for the flow field from the plug base to recompression; by subroutine BASE6 for the flow field downstream of recompression; and by the subroutine LIPSHK when a compressive turn results at separation.

Subroutine FLOW calls subroutine and function subprograms: BLAYER, CALC, CPB, PMANGL, PMTURN, SETUP, SSHAPE, STRLNE, SURF, and SURFP.

##### FORTRAN SYSTEM ROUTINES

Built-in FORTRAN functions ASIN, MOD, SIN, and SQRT are used.

##### CALLING SEQUENCE

The calling sequence is:

CALL FLOW (IREADS, NEPS, \$)

IREADS is an integer which determines the position of the characteristics solution in the nozzle.

$$NEPS = \begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$$

ACST4 - accuracy requirement in locating the j-streamline  
 AMBNT - Prandtl-Meyer angle (radians) at the point on the initial data line which is on the top contour  
 CSQ - square of the Crocco number on the plug just before separation  
 C3AS - square of the Crocco number adjacent to recompression  
 DELI - boundary layer thickness (inches) on the plug surface  
 DF1 - real number telling the number of vertical characteristic points  
 DI - non-dimensional distance along the initial data line  
 DLNU - change in Prandtl-Meyer angle between the shroud contour and the ambient conditions  
 DL2ST - boundary layer momentum thickness (inches)  
 DNU - incremental change in Prandtl-Meyer angle (radians) in going from the shroud contour to ambient conditions  
 DTC - incremental change in streamline angle (radians) in going from the plug surface to the near wake  
 EKJ - real number used in determining the J-subscript  
 EMRSRB - Mach number adjacent to recompression  
 EMULA - Mach angle of the flow on the plug just before separation  
 EM2A - Mach number of the flow adjacent to the near wake just after separation  
 EX - boundary layer velocity profile exponent  
 G - ratio of specific heats  
 GD - base bleed rate (lb/sec)  
 GDBL - equivalent base bleed rate due to the boundary layer  
 GDRAT - ratio of the base bleed to the sum of the base bleed plus the equivalent bleed due to the boundary layer  
 I - subscript  
 IBOUND - integer telling what type of base pressure solution is desired

ICORN - number of I-subscripts needed to move through the incremental change from the plug surface to the near wake  
 IEND - the last value of the I-subscript  
 ILOC - integer used in determining what portion of the flow field is being calculated (See Fig. IV-1)  
 ILOCN - equivalent to ILOC  
 IOPTR - integer which tells what type of initial data line read-in is used  
 IOPT1 - integer telling how many sets of characteristic matrices are needed in the near wake region  
 IS - starting value of the I-subscript  
 IST - starting value of the I-subscript  
 Ii - subscript  
 J - subscript  
 JB - subscript  
 JCORN - number of J-subscripts needed to move through the incremental change from the plug surface to the near wake  
 JT - subscript  
 K - subscript  
 K1 - subscript  
 LAM - plug temperature ratio,  $T_p/T_{o1}$   
 LOC - integer used in determining what portion of the flow field is being calculated (See Fig. I-3)  
 LOCN - equivalent to LOC  
 MAMB - Mach number corresponding to ambient conditions  
 MBLD - integer telling whether mass bleed is present  
 MESHPM - minimum number of incremental turns from the plug surface to the near wake  
 MI - integer used in determining whether I is odd or even

MIST - integer used in determining whether IST is odd or even  
 MK - integer used in determining whether K is odd or even  
 M1A - Mach number on the plug surface just before separation  
 N - denominator of the boundary layer profile exponent  
 NDF - final value of the I-subscript  
 NDI - maximum number of I-subscripts in the characteristics matrix  
 NDIM1 - NDI-1  
 NDIM2 - NDI-2  
 NDJ - maximum number of J-subscripts in the characteristics matrix  
 NOIPTS - number of initial data line points  
 NOPPTS - number of plug coordinates which are read in  
 NSPTS - number of recompression shock points  
 NTC - number of discrete turns in going from the plug surface to the  
       near wake  
 NTCMIN - minimum number of incremental turns from the plug surface to  
       the near wake  
 NU - array of Prandtl-Meyer angles (radians) for each point in the  
       characteristic matrix  
 NUAMB - Prandtl-Meyer angle (radians) corresponding to MAMB  
 NUI - array of Prandtl-Meyer angles (radians) for each point along  
       the initial data line  
 NU2A - Prandtl-Meyer angle (radians) of the flow adjacent to the near  
       wake immediately after separation  
 PA - ambient pressure ( $\text{lb/in}^2$ )  
 PO1 - chamber stagnation pressure ( $\text{lb/in}^2$ )  
 R - non-dimensional array of radial coordinates for each point in  
       the characteristic matrix  
 REDNUI - Prandtl-Meyer angle (radians) of the flow on the plug just be-  
       fore separation

RG - gas constant ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )  
 RI - array of non-dimensional radial coordinates at each point along the initial data line  
 RP - array of non-dimensional radial coordinates at each point along the plug surface  
 RPB - plug base radius (inches)  
 RSRB - wake radius ratio  
 R2 - non-dimensional base radius  
 S - array of entropies for each point in the characteristic matrix ( $\text{lb}_f\text{-ft}/\text{lb}_m\text{-}^\circ\text{R}$ )  
 SI - array of entropies at each point along the initial data line ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )  
 SINANG - the sine of the Mach angle, EMU1A  
 SOL3A - ratio of the Mach number at recompression to that just after separation, OR the streamline angle (radians) at recompression  
 S3A - the jet spread parameter  
 T - array of streamline angles (radians) at each point in the characteristic matrix  
 THET12 - total change in streamline angle (radians) from the plug surface to the near wake  
 THET3A - streamline angle (radians) just upstream of recompression  
 TI - array of streamline angles (radians) at each point along the initial data line  
 T01 - chamber stagnation temperature ( $^\circ\text{R}$ )  
 TP - array of streamline angles (radians) at each point along the plug surface  
 TS - streamline angle (radians) of the near wake  
 T2 - streamline angle (radians) of the last point on the plug surface  
 T3A - streamline angle (radians) just upstream of recompression  
 V3A - Prandtl-Meyer angle (radians) at the characteristic point which is just downstream of recompression

- X - array of non-dimensional axial coordinates for each point in the characteristic matrix
- XI - array of non-dimensional axial coordinates for each point along the initial data line
- XP - array of non-dimensional axial coordinates at each point along the plug surface
- XPF - non-dimensional axial location of the plug base
- XREC - non-dimensional axial location of recompression
- X2 - non-dimensional axial location of the plug base, OR the non-dimensional axial location of recompression

## SOLUTION METHOD

Redefine location integers. (See Fig. IV-1)

1.  $LOC = LOCN$
2.  $ILOC = ILOCN$

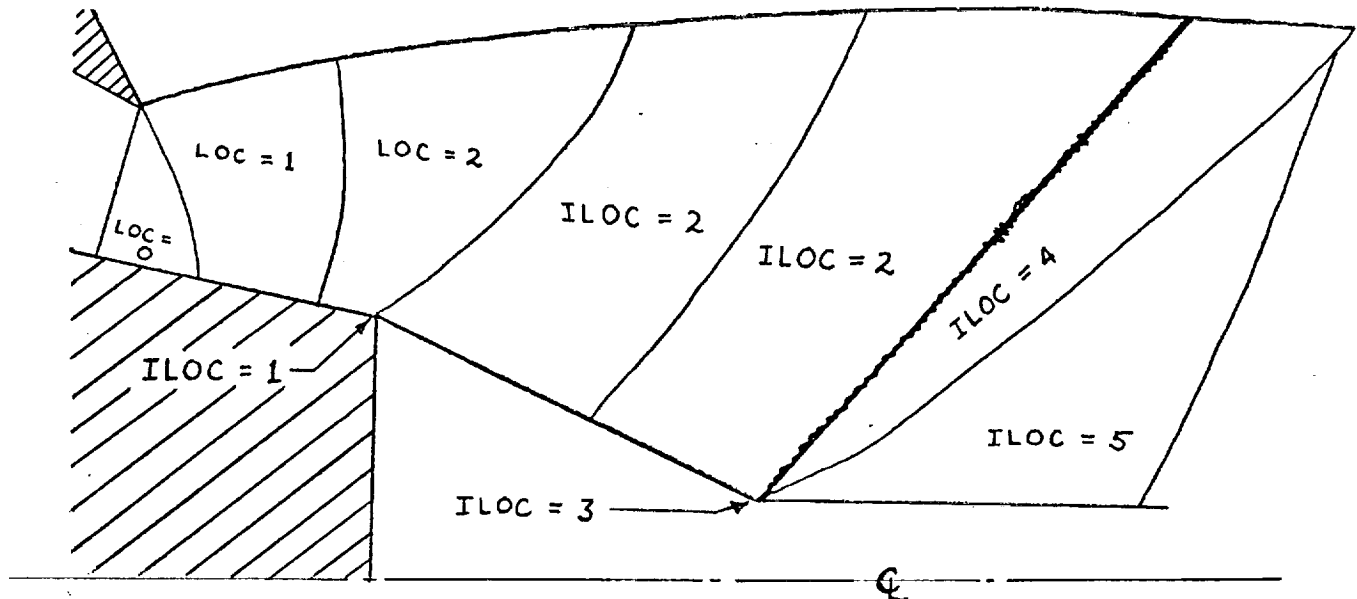


FIGURE IV - 1

Define calculational integers.

3.  $NDIM2 = NDI-2$
4.  $NDIM1 = NDI-1$
5.  $IPOT1 = 0$

The final value of the subscript I is set initially.

6.  $NDF = NDIM1$

The calculations now proceed to the various sections depending on the status and position of the flow field.  $IREADS = 0$  implies that the flow field over the plug is to be calculated;  $IREADS = 1$  is for the flow over the near wake;  $IREADS = 4$  is for the flow field downstream of recompression; and  $IREADS = 5$  is for the flow field containing a shock originating at separation.



7. If IREADS = 4, GO TO 164
8. If IREADS = 5, GO TO 89
9. If IREADS  $\neq$  0, GO TO 88

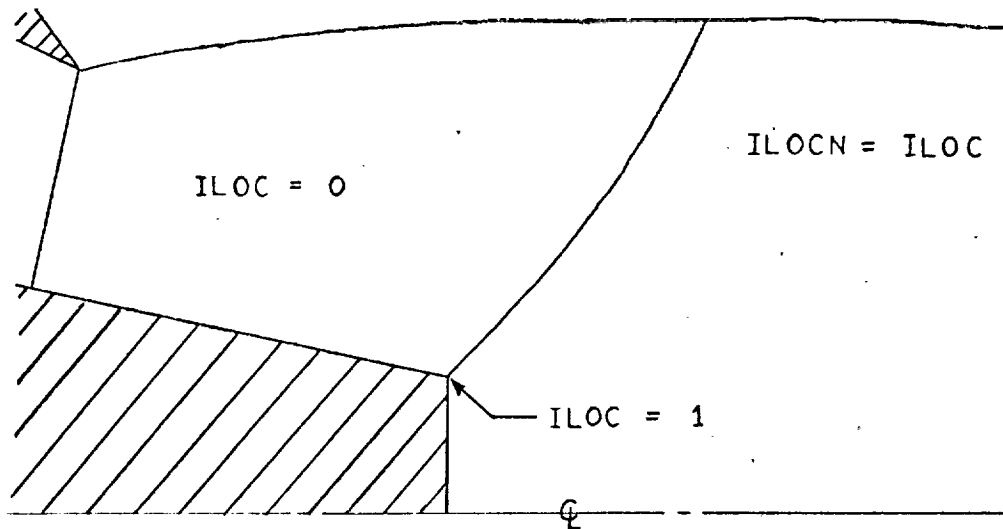


FIGURE IV - 2

Calculation of the flow field above the plug is done in the following section. (See Fig. IV-2)

10. DO 28 K = 2, NDJ

Set the last value of I

11. IEND = (2) (K) - 3

The term EKJ is used to determine the subscript J

12. EKJ = K + 1.10

The term ILOCN = 1 denotes that the plug base has been reached. The calculations, therefore, should stop there.

13. If ILOCN = 1, IEND = NDF-1

Move along a right-running characteristic from the initial data line.

14. DO 22 I = 1,IEND

Determine subscript J

15. EKJ = EKJ-0.50

16. J = EKJ

Determine whether the subscript I is odd or even.

17. MI = MOD(I,2)

Calculate an interior flow field point.

18. If MI = 0, GO TO 21

The subscript I is odd.

19. CALL CALC

20. GO TO 22

The subscript I is even

21. CALL CALC

22. CONTINUE

Check to see if the plug base has been reached.

23. If ILOCN = 1, GO TO 28

The base has not been reached.

24. I = IEND + 2

Calculate the plug surface point.

25. CALL SURFP

Check to see if plug base has been reached with this point.

26. If ILOCN = 0, GO TO 28

The plug base has been reached. Set last value of I.

27. NDF = I

28. CONTINUE

The flow field is now completed along and below a right-running characteristic originating at the intersection of the initial line and the shroud contour. The remainder of the flow field will now be calculated. The solution will proceed along a right-running characteristic originating at the external free jet boundary. First calculate the Prandtl-Meyer (P-M) angle corresponding to the ambient pressure.

$$29. \quad \text{NUAMB} = \text{PMANGL} (\text{MAMB}, \text{G})$$

Calculate the change in P-M angle required to expand about this corner.

$$30. \quad \text{DLNU} = \text{NUAMB} - \text{NUI}(\text{NOIPTS})$$

Determine the number of characteristic points available to expand to the ambient pressure.

$$31. \quad \text{DF1} = (\text{NDF} - 1.0) / 2.0$$

Calculate the increment of P-M angle at each location.

$$32. \quad \text{DNU} = \text{DLNU} / \text{DF1}$$

Redefine the exit P-M angle.

$$33. \quad \text{AMBNT} = \text{NUI}(\text{NOIPTS})$$

Proceed with the characteristics calculations along a right-running wave.

$$34. \quad \text{DO } 51 \text{ K} = 3, \text{ NDF}, 2$$

Determine the subscript on the last point and the subscript J on the boundary surface.

$$35. \quad \text{IEND} = \text{NDF} - 1$$

$$36. \quad \text{J} = \text{NDJ}$$

Increment the P-M angle and check to make sure it has not exceeded the value corresponding to ambient conditions.

$$37. \quad \text{AMBNT} = \text{AMBNT} + \text{DNU}$$

$$38. \quad \text{If } \text{AMBNT} > \text{NUAMB}, \text{ AMBNT} = \text{NUAMB}$$

If the initial data line was read in along a characteristic line (IOPTR = 2), the above incrementation has already been completed in Subroutine TPNZZL and is not repeated here again.

39. If IOPTR = 2, AMBNT = NUAMB

Calculate the position and flow field properties along the constant pressure boundary.

40. CALL CPB

Make sure the solution is not at the last point.

41. If K = NDF, GO TO 51.

Now proceed along the right-running characteristic originating at the above-calculated boundary point.

42. EKJ = NDJ + 1.10

43. DO 51 I = K, IEND

Determine the subscript J.

44. EKJ = EKJ - 0.50

45. J = EKJ

See whether I is odd or even.

46. MI = MOD(I,2)

Calculate interior flow field point.

47. If MI = 0, GO TO 50

The subscript I is odd.

48. CALL CALC

49. GO TO 51

The subscript I is even.

50. CALL CALC

51. CONTINUE

If the plug base has not been reached, reincrement the characteristic starting line. SETUP returns to statement number 54.

52. If ILOCN = 1, CALL SETUP

The plug base has been reached.

53. GO TO 57

Redefine location integers.

54.  $LOC = LOC + 1$

55.  $LOCN = LOC$

Repeat above process until the plug base is reached.

56. GO TO 6

The plug base has been reached. Redefine the P-M angle at the plug tip upstream of separation.

57.  $REDNUI = NU(NDF,1)$

Determine the corresponding Mach number,  $M1A$

58. CALL PMTURN

Define the position and streamline angle at the plug base.

59.  $X2 = XPF$

60.  $R2 = 1.000$

61.  $T2 = TP(NOPPTS)$

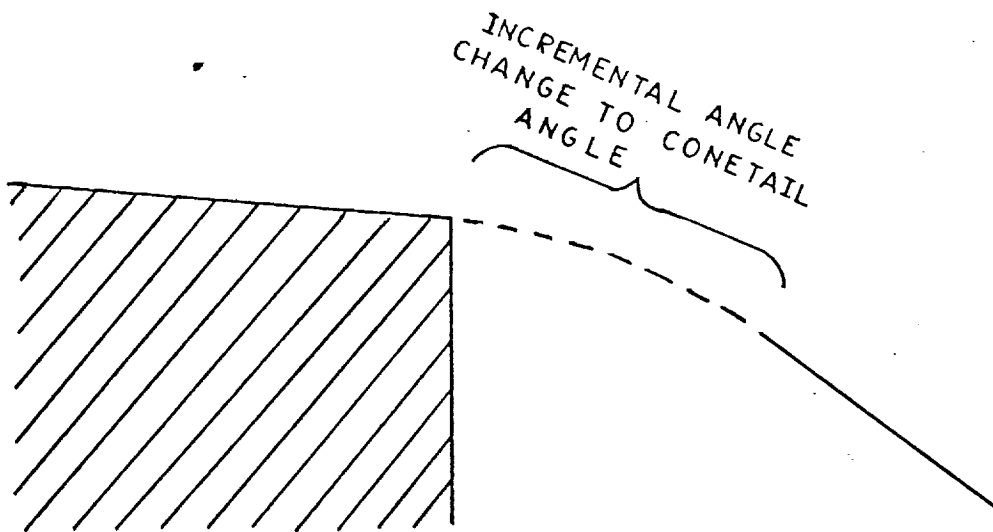


FIGURE IV - 3

Determine the number of discrete steps into which the expansion about the plug is divided. (See Fig. IV-3)

62.  $MESH_{PM} = 2.50 + (T_2)(5.729578)$

63. If  $MESH_{PM} > 2$ ,  $MESH_{PM} = 2$

Calculate streamlines contained in the characteristic matrix.

64. CALL STRLNE

Redefine location integers.

65.  $LOC = IOC + 1$

66.  $LOCN = LOC$

The characteristics solution up to the plug base is completed. The initial data line for the iterative part of the solution is not set.

67. DO 72 J = 1, NDJ

68.  $XI(J) = X(NDF, J)$

69.  $RI(J) = R(NDF, J)$

70.  $NUI(J) = NU(NDF, J)$

71.  $TI(J) = T(NDF, J)$

72.  $SI(J) = S(NDF, J)$

73.  $NOPTS = NDJ$

Determine the linear distance along the initial profile of each point measured from the plug tip.

74.  $DI(1) = 0$

75. DO 76 I = 2, NOPTS

76.  $DI(I) = DI(I-1) + \sqrt{(XI(I) - XI(I-1))^2 + (RI(I) - RI(I-1))^2}$

Determine the Mach angle of the flow at the corner.

77.  $EMULA = \text{ASIN}(1/M1A)$

78.  $SINANG = \text{SIN}(EMULA)$

Set the minimum number of turns for the flow expanding around the plug tip.

79.  $NTCMIN = 2$

Calculate the actual number of turns. (See Fig. IV-3)

80.  $NTC = (NTCMIN) (MESHPM)$

Determine how far the corner effects will permeate into the flow field.

81.  $JCORN = NTC + 1$

82.  $ICORN = (2) (JCORN) - 1$

Calculate the momentum thickness and the equivalent base bleed of a boundary layer if one exists.

83. If  $MBLD > 1$ , CALL BLAYER

The term GDRAT is the ratio of base bleed to the sum of base bleed and equivalent base bleed due to the boundary layer.

84.  $GDRAT = 1.00$

85. If  $MBLD > 2$ , RETURN

86.  $GDRAT = GD / (GD + GDBL)$

Combine both base bleed terms into one.

87.  $GD = GD + GDBL$

Return to calling program.

The next part of the subroutine flow calculates the flow field by the method of characteristics up to recompression.

Reset position integer.

88.  $ILOCN = 1$

Set the last point.

89.  $NDF = NDI - 1$

The variable IOPT1 = 0 means that this region begins with the initial

data line, while  $IOPT1 > 1$  implies that the characteristic matrix begins downstream of this point.

90. If  $IOPT1 > 0$ ,  $ILOCN = 2$

91.  $ILOC = ILOCN$

The characteristics solution will move along right-running characteristics from the initial data line. (Fig. IV-4)

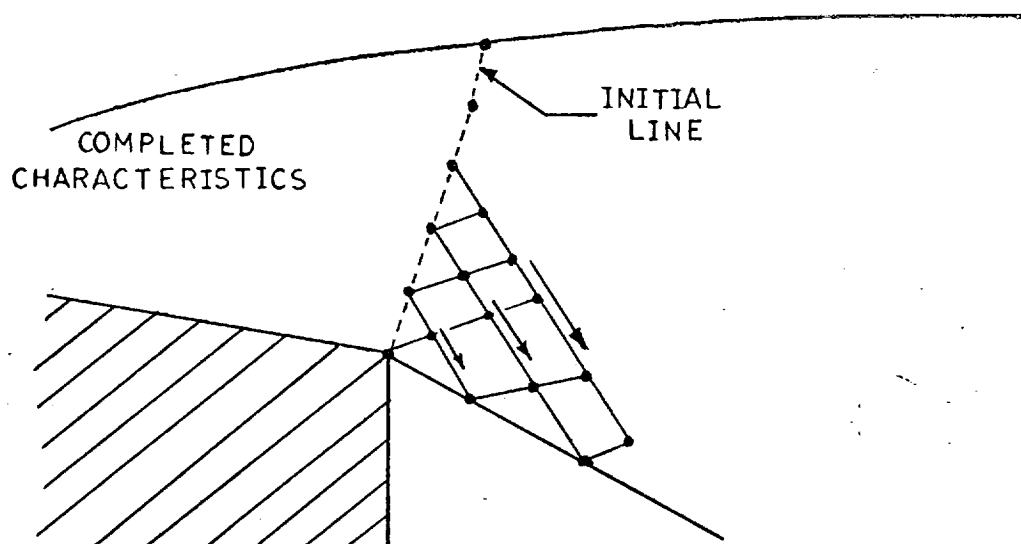


FIGURE IV - 4

92. DO 137 K = 2,NDJ

See whether K is odd or even.

93.  $MK = \text{MOD}(K, 2)$

Determine last value of subscript I.

94.  $IEND = (2)(K) - 3$

The term EKJ is used to determine the subscript J.

95.  $EKJ = K + 1.10$

96. If  $ILOC = 1$  and K is even,  $EKJ = (K/2) + 1.50$

97. If  $ILOC = 1$  and K is odd,  $EKJ = (K/2) + 2.10$



Check to see whether recompression has been encountered.

98. If ILOCN = 3, IEND = NDF-1

The initial data line for ILOC = 1 is a left-running characteristic from the plug tip. For ILOC > 1, the initial data line is a non-characteristic line. Set the initial value of I.

99. IS = 1

100. If ILOC = 1, IS = K

Move along the right-running characteristic.

101. DO 110 I = IS, IEND

Make sure extra points are not calculated.

102. If IS > IEND, GO TO 110

Determine index J.

103. EKJ = EKJ - 0.50

104. J = EKJ

See whether I is odd or even.

105. MI = MOD(I,2)

106. If MI = 0, GO TO 109

The index I is an odd number.

107. CALL CALC

108. GO TO 110

I is an even number

109. CALL CALC

110. CONTINUE

If recompression has been encountered previously, the boundary calculations are omitted.

111. If ILOC = 3, GO TO 137

The index I for the boundary point is determined.

112.  $I = IEND + 1$

A solid boundary solution is used if a conetail base pressure solution ( $IBOUND = 1$ ) is used; also it is used while the flow is still expanding about the plug tip. Otherwise the constant pressure boundary solution is used.

113. If  $IBOUND = 2$ , and  $I > ICORN$ , and  $ILOCN = 1$ , THEN GO TO 119

A solid surface (conetail) is indicated.

Calculate the streamline angle of the near wake at this point.

114.  $TS = T2 - THET12$

Calculate the characteristic point on the solid surface.

115. CALL SURF

Determine if the last point in the expansion has been reached. If so, redefine the P-M angle.

116. If  $ILOCN = 1$ , and  $(I + 1) = ICORN$ ,  $NU2A = NU(I+1,1)$

Determine if recompression has been encountered.

117. If  $R(I+1,1) > RSRB$ , GO TO 122

118. GO TO 137

Calculate the constant pressure boundary solution.

119. CALL CPB

See if recompression has been encountered.

120. If  $(RI+1,1) > RSRB$ , GO TO 122

121. GO TO 137

Recompression has occurred.

122.  $ILOCN = 3$

Determine the Mach number ( $M2A$ ) corresponding to  $NU2A$ .

123. CALL PMTURN

Interpolate for all variables at recompression, and redefine each.

124.  $NU(I+1,1) = NU(I-1,1) + ((NU(I+1,1) - NU(I-1,1)) / (R(I+1,1) - R(I-1,1))) (RSRB - R(I-1,1))$

125.  $V3A = NU(I+1,1)$

126.  $X(I+1,1) = (I-1,1) + ((X(I+1,1) - X(I-1,1)) / (R(I+1,1) - R(I-1,1))) (RSRB - R(I-1,1))$

127.  $XREC = X(I+1,1)$

128.  $R(I+1,1) = RSRB$

129.  $T(I+1,1) = T(I-1,1) + ((T(I+1,1) - T(I-1,1)) / (R(I+1,1) - R(I-1,1))) (RSRB - R(I-1,1))$

130.  $T3A = T(I+1,1)$

131.  $S(I+1,1) = S(I-1,1) + ((S(I+1,1) - S(I-1,1)) / (R(I+1,1) - R(I-1,1))) (RSRB - R(I-1,1))$

132.  $S3A = S(I+1,1)$

Determine the Mach number at recompression, EMRSRB

133. CALL PMTURN

For a conetail solution, SOL3A is a ratio of Mach numbers; for a constant pressure boundary solution, SOL3A is the streamline angle at compression.

134. If IBOUND = 1, then  $SOL3A = EMRSRB/EM2A$

135. If IBOUND = 2, then  $SOL3A = T3A$

Set the last point.

136.  $NDF = I + 1$

137. CONTINUE

Now calculate the flow field to the non-characteristic line originating at recompression.

138. DO 159 K = 3, NDF, 2

Determine last value of subscript I

139.  $IEND = NDF = 1$

For ILOC = 1 (the first region of characteristics above the near

wake region) the initial line is along a left-running characteristic.

140. If  $ILOC = 1$ , then GO TO 145

Set the J subscript on the external free jet (constant pressure) boundary.

141.  $J = NDJ$

Calculate the boundary point.

142. CALL CPB

The term EKJ is used to determine the J subscript.

143.  $EKJ = NDJ + 1.10$

Determine the starting value of I

144.  $IST = K$

Do not calculate more points than necessary.

145. If  $K = NDF$ , then GO TO 159

146. If  $ILOC = 1$ , then  $IST = NDJ + (K/2)$

See whether the starting value of I is odd or even.

147.  $MIST = MOD(IST, 2)$

Determine EKJ for  $ILOC = 1$

148. If  $ILOC = 1$  and IST is even, then  $EKJ = (IST/2) + 1.60$

149. If  $ILOC = 1$  and IST is odd, then  $EKJ = (IST/2) + 2.10$

Begin calculating along a right-running characteristic.

150. DO 159 I = IST, IEND

Make sure extra points are not calculated.

151. If  $IST > IEND$ , GO TO 160

Determine J subscript.

152.  $EKJ = EKJ - 0.50$

153.  $J = EKJ$

See whether I is odd or even.

154.  $MI = \text{MOD}(I, 2)$

155. If  $MI = 0$ , then GO TO 158

The subscript I is an odd number.

156. CALL CALC

157. GO TO 159

I is an even number.

158. CALL CALC

159. CONTINUE

Redefine location integers

160.  $ILOC = ILOCN$

The term  $IPOT1$  tells how many sets of characteristic matrices are required to describe the flow field above the near wake region.

161.  $IOPT1 = IOPT1 + 1$

Set up the initial data line for the next set of characteristic calculations. The subroutine SETUP returns to statement number 90 if recompression has not been reached. Otherwise a normal return is used.

162. CALL SETUP

163. Return to calling program.

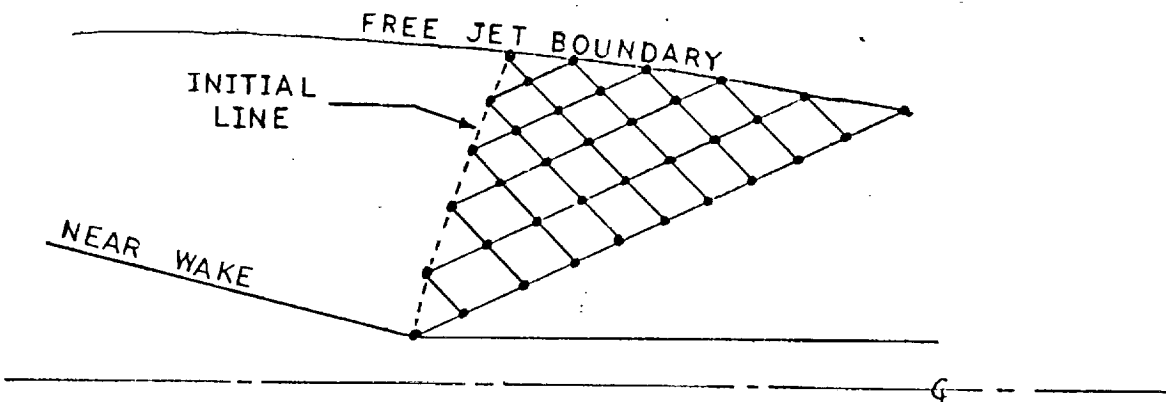


FIGURE IV - 5

This next section performs the method of characteristics solution to a left-running characteristic originating at recompression. The recompression shock wave is ignored for the present. First, determine the last value of I. (See Fig. IV-5)

164. IEND = 2

Begin calculations

165. DO 180 K = 2, NDJ

The term EKJ is used to determine the index J

166. EKJ = K + 0.60

Move along a right-running characteristic.

167. DO 178 I = 2, IEND

Calculate J subscript

168. EKJ = EKJ - 0.50

169. J = EKJ

Determine whether I is odd or even.

170. MI = MOD(I,2)

171. If MI = 0, then GO TO 175

Only one call to the subroutine CALC is used; therefore the J index must be varied depending on whether the index I is odd or even.

172. JT = J + 1

173. JB = J

174. GO TO 177

175. JT = J

176. JB = J-1

Determine the I subscript.

177. I1 = I-1

Calculate the interior point.

178. CALL CALC

Redefine the last value of I

179. IEND = IEND + 1

180. CONTINUE

Now finish the characteristic matrix. The solution begins at the external free jet boundary and moves along right-running characteristics to the left-running characteristic originating at recompression.

181. DO 199 K = 3, NDIM1,2

Set upper boundary index

182. J = NDJ

Calculate upper boundary point.

183. CALL CPB

Continue moving along a right-running characteristic.

184. EKJ = J + 0.60

185. K1 = K + 1

186. DO 197 I = K1, IEND

Determine J subscript.

187. EKJ = EKJ - 0.50

188. J = EKJ

See whether I is odd or even.

189. MI = MOD(I,2)

190. If MI = 0, GO TO 194

Again define J indices.

191. JT = J + 1

192. JB = J

193. GO TO 196

194. JT = J

195.  $JB = J - 1$

Define I subscript.

196.  $Il = I - 1$

Calculate interior point.

197. CALL CALC

Redefine last point.

198.  $IEND = IEND + 1$

199. CONTINUE

Redefine location integer.

200. If  $IREADS = 4$ , then  $ILOCN = 5$

Determine the recompression shock shape if recompression has been reached.

201.  $IREADS = 3$ , then CALL SSHAPE

202. Return.



## V. SUBROUTINE SSHAPE

## V. SUBROUTINE SSHAPE

Subroutine SSHAPE calculates the shape of the recompression shock. An initial estimate of the shock shape is made and this is modified as the method of characteristics solution downstream of the shock is calculated.

### COMMON BLOCKS

COMMON blocks AMB, CORNER, GAS, PARAM, POLIP, SIZE, SOLBLK, STRBLK, STRL, THETBK, and TPN are used.

### TPNZZL SUBROUTINES

Subroutine FLOW or LIPSHK calls SSHAPE.

SSHAPE calls subroutines and functions CALC, OSHOCK, PMANGL, PMTURN, SHOCK, STRLNE, SURF, SURFSK, and TAB.

### FORTTRAN SYSTEM ROUTINES

Built-in FORTRAN functions ASIN, ATAN, MOD, SIN, SQRT, and TAN are used.

### CALLING SEQUENCE

The calling sequence is:

CALL SSHAPE (NSPTS, NEPS, XREC, SW, NDF)

NSPTS is the number of shock points along the shock.

$$\text{NEPS} = \begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$$

XREC is the axial location of recompression.

SW is the shock wave angle at recompression (radians).

NDF is the final value of the axial characteristic subscript occurring at recompression.

AMBM - Mach number corresponding to ambient conditions  
 ANGCH - the angle (radians) of the left-running characteristic  
 COTDEL - the cotangent of the change in streamline angle across the shock  
 C3AS - square of the Crocco number just upstream of recompression  
 D - distance along the shock wave (non-dimensional)  
 DD - non-dimensional incremental distance along the shock  
 DDEL - minimum change in streamline angle (radians) across the shock wave  
 DEL - change in streamline angle (radians) across the shock  
 DELTA - change in streamline angle (radians) across the shock at recompression; change in streamline angle across the shock  
 DFMY - difference in downstream Mach number as calculated from the characteristics solution and the shock equations  
 DJM1 - real number value of NDJ-1  
 DNU - change in Prandtl-Meyer angle (radians) between the last downstream shock point and ambient conditions  
 DRS - density ratio across the shock  
 DS - non-dimensional distance along the shock  
 DSR - the ratio of the change in entropy across the shock to the gas constant  
 DTC - the incremental change in streamline angle (radians) between the plug surface and the near wake  
 DTOT - total non-dimensional distance along the shock wave  
 EKJ - real number used in determining the J-subscript  
 EMU2 - Mach angle (radians) corresponding to a characteristic point  
 EMX - Mach number on the upstream side of the shock  
 EM1 - Mach number on the upstream side of the shock  
 EM2 - Mach number corresponding to a characteristic point downstream of the shock

G        - ratio of specific heats  
 I        - subscript  
 IFIN    - the final I-subscript  
 IINT    - the value of the I-subscript of the characteristic point which  
          intersects the shock wave  
 IL       - subscript  
 IP1     -  $I + 1$   
 IS       - subscript  
 IST     - starting value of the I-subscript  
 IS1     - starting value of the I-subscript  
 J        - subscript  
 JS       - subscript  
 JST     - starting value of the J-subscript  
 JS1     - subscript  
 J1       - subscript  
 K        - subscript  
 K1       - subscript  
 L        - subscript  
 LS       - subscript  
 LSHK    - integer denoting whether a "lip shock" is present  
 LTP     - subscript  
 MAMB    - Mach number corresponding to ambient conditions  
 MI       - integer used in determining whether the I-subscript is odd or  
          even  
 MINT    - upstream Mach number at the intersection of the characteristic  
          and the shock wave  
 MS       - Mach number on the upstream side of the shock

MSTR - array of Mach numbers along the streamlines  
 MY - Mach number on the downstream side of the shock  
 M3A - Mach number on the near wake just upstream of recompression  
 NCOUNT - iteration counter used in matching the characteristic solution  
           with the shock solution  
 NDI - maximum number of I-subscripts in the characteristic matrix  
 NDIM1 - NDI-1  
 NDIM2 - NDI-2  
 NDJ - maximum number of J-subscripts in the characteristic matrix  
 NDJM1 - NDJ-1  
 NINTS - number of characteristic-shock wave intersections  
 NOSPTS - number of streamline points  
 NOSTRL - number of streamlines  
 NS - Prandtl-Meyer angle (radians) at the first upstream character-  
       istic point from the shock  
 NSKPTS - number of shock points  
 NSL1 - number of streamlines - 1  
 NS1 - Prandtl-Meyer angle (radians) at the second upstream character-  
       istic point from the shock  
 NTC - number of discrete turns in going from the plug surface to the  
       near wake  
 NU - array of Prandtl-Meyer angles (radians) at each point in the  
       characteristic matrix  
 NUAMB - Prandtl-Meyer angle (radians) corresponding to MAMB  
 NUINT - Prandtl-Meyer angle (radians) on the downstream side of the  
       shock at the intersection of the characteristic and the  
       shock wave  
 NUSX - array of Prandtl-Meyer angles (radians) on the upstream side  
       of the shock  
 NUSX1 - array of Prandtl-Meyer angles (radians) at the second point  
       upstream of the shock

NUSY - array of Prandtl-Meyer angles (radians) on the downstream side of the shock  
 NUX - Prandtl-Meyer angle (radians) on the upstream side of the shock at the intersection of the characteristic and the shock wave  
 NUY - Prandtl-Meyer angle (radians) on the downstream side of the shock at the intersection of the characteristic and the shock wave  
 PA - ambient pressure ( $\text{lb/in}^2$ )  
 POLP - stagnation pressure downstream of the "lip shock" ( $\text{lb/in}^2$ )  
 POR - ratio of stagnation pressures across a shock  
 POX - stagnation pressure upstream of the shock ( $\text{lb/in}^2$ )  
 POY - stagnation pressure downstream of the shock ( $\text{lb/in}^2$ )  
 POYPOX - ratio of stagnation pressures across a shock  
 POL - chamber stagnation pressure ( $\text{lb/in}^2$ )  
 PR - static pressure ratio across a shock  
 PYPX - static pressure ratio across a shock  
 R - array of non-dimensional radial coordinates at each point in the characteristic matrix  
 RATIO - ratio of distances  
 RB - non-dimensional radial coordinate of the streamline at its intersection with the shock  
 RDIM - dimensional radial coordinate (inches) - used for output  
 RG - gas constant ( $\text{ft-lb/lb}_m\text{-}^\circ\text{R}$ )  
 RINT - non-dimensional radial coordinate at the intersection of the shock wave and the characteristic  
 RP - array of non-dimensional radial coordinates for each plug point  
 RPB - plug base radius (inches)  
 RQ - non-dimensional radial location of the characteristic point which intersects the shock wave

RS - the non-dimensional radial coordinate of the point on the upstream side of the shock

RSRB - wake radius ratio

RSTR - non-dimensional array of radial coordinates of the streamlines

RSX - array of non-dimensional radial coordinates of the point on the upstream side of the shock

RSX1 - array of non-dimensional radial coordinates of the second point upstream of the shock

RSY - array of non-dimensional radial coordinates of the point on the downstream side of the shock

RS1 - array of non-dimensional radial coordinates of the second point upstream of the shock

RYRX - density ratio across the shock

R2 - non-dimensional base radius, OR wake radius ratio

R2P - non-dimensional base radius

S - array of the entropies at each point in the characteristic matrix ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )

SA - shock wave angle (radians)

SACALC - shock wave angle (radians) at the intersection of a streamline and the shock wave

SAR - shock wave angle (radians)

SAS - array of shock wave angles (radians)

SINT - entropy at the intersection of the shock wave and the characteristic ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )

SLB - slope of the free jet boundary at the intersection with the shock wave

SLCH - slope of the characteristic

SLSK - slope of the shock

SLST - slope of the streamline

SOL3A - ratio of the Mach number just upstream of recompression to that

just after separation, OR the streamline angle (radians)  
at recompression

SS - array of entropies on the upstream side of the shock

SSX - array of entropies on the upstream side of the shock

SSX1 - array of entropies on the second point upstream of the shock

SSY - array of entropies on the downstream side of the shock

SS1 - array of entropies on the second point upstream of the shock

SWAD - shock wave angle in degrees (used for output)

SWAR - shock wave angle in radians

SX - array of entropies on the upstream side of the shock

SY - array of entropies on the downstream side of the shock

S3NT - the entropy on the downstream side of the shock at the intersection of the characteristic and the shock wave

T - array of streamline angles (radians) at each point in the characteristics matrix

THET12 - the change in streamline angle (radians) from the plug surface to the near wake

THET3A - streamline angle (radians) just upstream of recompression

TINT - streamline angle (radians) at the intersection of the shock wave and the characteristic

TO1 - chamber stagnation temperature ( $^{\circ}$ R)

TP - array of streamline angles (radians) at each point on the plug surface

TRS - static temperature ratio across the shock

TS - array of streamline angles (radians) on the upstream side of the shock

TSTR - array of streamline angles (radians) along a streamline

TSX - array of streamline angles (radians) on the upstream side of the shock



TSX1 - array of streamline angles (radians) on the second point on the upstream side of the shock

TSY - array of streamline angles (radians) the downstream side of the shock

TS1 - array of streamline angles (radians) on the second point on the upstream side of the shock

TW - streamline angle (radians) of the far wake

TX - streamline angle (radians) on the upstream side of the shock at the point where the characteristic intersects the shock

TXD - streamline angle in degrees on the upstream side of the shock (used for output)

TY - streamline angle (radians) on the downstream side of the shock at the point where the characteristic intersects the shock

TYD - streamline angle in degrees on the downstream side of the shock (used for output)

TYG - an estimate of the downstream streamline angle (radians) used in the iteration procedure

TYTX - static temperature ratio across the shock

X - array of non-dimensional axial coordinates at each point in the characteristic matrix

XB - non-dimensional axial coordinate of a streamline and its intersection with a shock wave

XCALC - non-dimensional axial coordinate of streamline and its intersection with a shock wave

XDIM - axial coordinate in inches (used for output)

XINT - non-dimensional axial coordinate at the intersection of the shock wave and the characteristic

XP - array of non-dimensional axial coordinates at each point on the plug surface

XPF - non-dimensional axial location of the plug base

XQ - non-dimensional axial coordinate of the characteristic point which intersects the shock wave

XS      - non-dimensional axial coordinate of the point on the upstream side of the shock (an array)

XSTR   - non-dimensional axial coordinate of the point on a streamline (an array)

XSX    - non-dimensional axial coordinate of the point on the upstream side of the shock (an array)

XSX1   - array of non-dimensional axial coordinates of the second point upstream of the shock

XSX    - array of non-dimensional axial coordinates at the points on the downstream side of the shock

XS1    - array of non-dimensional axial coordinates of the second point upstream of the shock

X2      - non-dimensional location of the plug base, OR the non-dimensional location of recompression

X2P    - non-dimensional axial location of the plug base

## SOLUTION METHOD

Redefine the stagnation pressure.

1.  $POX = POI$

2. If  $LSHK = 1$ , then  $POX = POLP$

Calculate streamlines in characteristic mesh.

3. CALL STRLNE

Set up the initial estimate of the shock shape. First prescribe a minimum turn angle through the shock.

4.  $DDEL = 0.090$

Determine calculational integers.

5.  $NDIM1 = NDI - 1$

6.  $NDIM2 = NDI - 2$

7.  $NDJM1 = NDJ - 1$

8.  $DJM1 = NDJM1$

Determine the position and variables of the first point of the shock wave.

9.  $XSX(1) = XREC$

10.  $RSX(1) = RSRB$

The variable M3A is the Mach number at recompression.

11.  $M3A = \sqrt{(2/(G-1)) (C3AS/(1-C3AS))}$

12.  $NUSX(1) = PMANGL(M3A,G)$

13.  $TSX(1) = - THET3A$

14.  $SSX(1) = SW$

The variable DELTA is the change in streamline angle through the recompression shock.

15.  $DELTA = THET3A$

Calculate the Mach number at recompression.

16. CALL PMTURN

Calculate the shock wave angle at recompression.

17. CALL SHOCK

Determine additional properties across the shock at recompression.

18.. CALL OSHOCK

Calculate the entropy along the far wake.

19.  $SW = SW + (DSR)(RG)$

Determine the position and flow field properties downstream of the shock at recompression.

20.  $XSX(1) = XSX(1)$

21.  $RSX(1) = RSX(1)$

22.  $TSX(1) = 0.00$

23.  $NUSX(1) = PMANGL(MY,G)$

24.  $SSX(1) = SW$

The variable K here is used to denote the number of shock wave points. K is incremented later.

25.  $K = 2$

Set up the initial estimate of shock shape. The shock should be in the characteristic matrix. (See Fig. V-1)

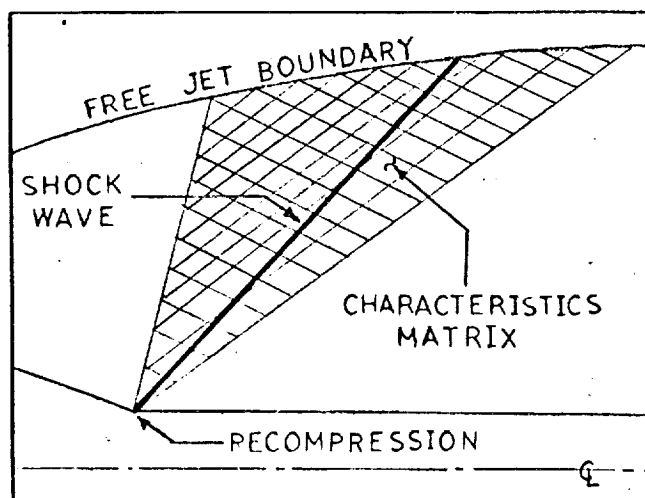


FIGURE V-1

26. DO 78 L = 2,NDJ

Determine the Mach number and shock wave angle of the (K-1) point.

27. CALL PMTURN

28. CALL SHOCK

Calculate the shock wave angle with respect to the axis.

29. SA(K-1) = SWAR + TSX(K-1)

Determine the slope of the shock wave.

30. SLSK = TAN(SA(K-1))

The term EKJ is used to increment the subscript J

31. EKJ = L + 0.60

The following section moves along a right-running characteristic in the calculated characteristic matrix until an intersection with the given portion of the shock wave is found.

32. DO 77 I = 2,L

Determine the J subscript.

33. EKJ = EKJ - 0.50

34. J = EKJ

Watch out for an incomplete characteristic matrix.

35. If  $X(I,J) < 10^{-6}$ , then GO TO 78

See if I is odd or even.

36. MI = MOD(I,2)

37. If MI = 0, GO TO 49

The subscript I is odd. Calculate the slope of the characteristic and its axial intersection with the shock.

38. SLCH = (R(I,J) - R(I-1,J+1))/(X(I,J) - X(I-1,J+1))

39. XINT = (R(I-1,J+1) - RSX(K-1) + (SLSK)(XSX(K-1)) -  
(SLCH)(X(I-1,J+1)))/(SLSK-SLCH)

See if the intersection is contained between the I and the I-1 points along the right-running wave.

40. If  $X(I-1, J+1) > XINT$ , then GO TO 78

41. If  $XINT > X(I, J)$ , GO TO 77

42. If  $XINT < XSX(K-1)$ , GO TO 78

There is a legitimate intersection. Save the upstream point before the intersection. (See Fig. V-2)

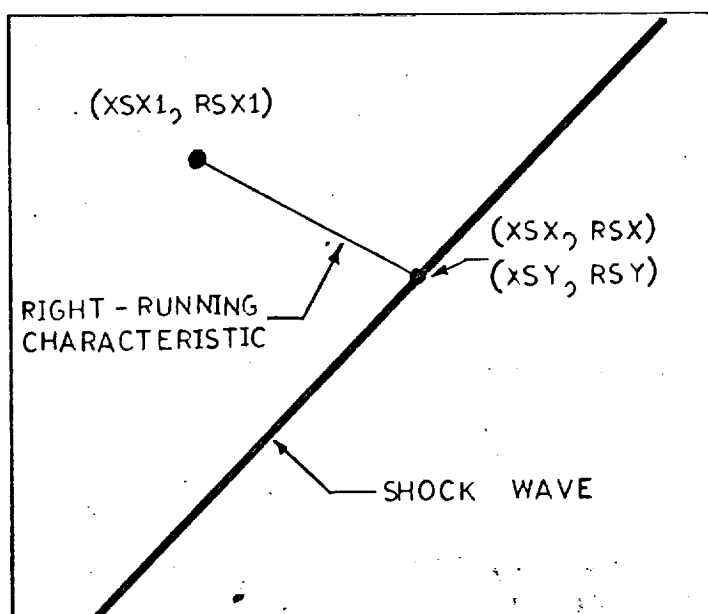


FIGURE V-2

43.  $XSX1(K) = X(I-1, J+1)$

44.  $RSX1(K) = R(I-1, J+1)$

45.  $NUSX1(K) = NU(I-1, J+1)$

46.  $TSX1(K) = T(I-1, J+1)$

47.  $SSX1(K) = S(I-1, J+1)$

48. GO TO 59

The subscript I is even. Calculate the slope of the characteristic.

49.  $SLCH = (R(I, J) - R(I-1, J)) / (X(I, J) - X(I-1, J))$

Calculate the axial location of the intersection.

$$50. \quad XINT = (R(I-1,J) - RSX(K-1) + (SLSK)(XSX(K-1)) - (SLCH)(X(I-1,J)))/(SLSK - SLCH)$$

See if the intersection is contained between the I and the I-1 points.

51. If  $X(I-1,J) > XINT$ , then GO TO 78

52. If  $XINT > X(I,J)$ , GO TO 77

53. If  $XINT < XSX(K-1)$ , GO TO 78

Save the characteristic point immediately upstream of this intersection.

$$54. \quad XSX1(K) = X(I-1,J)$$

$$55. \quad RSX1(K) = R(I-1,J)$$

$$56. \quad NUSX1(K) = NU(I-1,J)$$

$$57. \quad TSX1(K) = T(I-1,J)$$

$$58. \quad SSX1(K) = S(I-1,J)$$

Calculate the upstream coordinates and variables along the shock.  
A linear interpolation is used. (See Fig. V-2)

$$59. \quad XSX(K) = XINT$$

$$60. \quad RSX(K) = (SLCH)(XINT - X(I,J)) + R(I,J)$$

Calculate a ratio of distances.

$$61. \quad \text{RATIO} = (XINT - XSX1(K))/(X(I,J) - XSX1(K))$$

Calculate remaining variables.

$$62. \quad TSX(K) = TSX1(K) + (\text{RATIO})(T(I,J) - TSX1(K))$$

$$63. \quad NUSX(K) = NUSX1(K) + (\text{RATIO})(NU(I,J) - NUSX1(K))$$

$$64. \quad SSX(K) = SSX1(K) + (\text{RATIO})(S(I,J) - SSX1(K))$$

Now calculate the downstream location at the intersection.

$$65. \quad XSY(K) = XSX(K)$$

$$66. \quad RSY(K) = RSX(K)$$

Calculate the downstream streamline angle, but make sure a compressive turn results.

67.  $TSY(K) = 0.00$

68. If  $TSX(K) > 0.0$ ,  $TSY(K) = TSX(K) + 1/57.2957795$

Determine the total change in streamline angle across the shock.

69.  $DEL = TSY(K) - TSX(K)$

Make sure this angle is not too small.

70. If  $DEL < DDEL$ , then  $TSY(K) = TSX(K) + DDEL$

Calculate additional variables across the shock at the intersection.

71. CALL PMTURN

72. CALL OSHOCK

73.  $NUSY(K) = PMANGL(MY,G)$

74.  $SSY(K) = (DSR)(RG) + SSX(K)$

Calculate the total change in streamline angle.

75.  $DELTA = TSY(K) - TSX(K)$

Increment the number of shock intersections.

76.  $K = K + 1$

77. CONTINUE

78. CONTINUE

The above procedure is repeated; the characteristics originate now along the external free jet boundary.

First set the last value of the subscript I.

79.  $IFIN = NDJ$

Now begin moving along right-running characteristics.

80. DO 132 L = 4, NDIM2,2

Determine the shock wave angle at the (K-1) shock point.



81. CALL PMTURN

82. CALL SHOCK

83.  $SA(K-1) = SWAR + TSX(K-1)$

Calculate the slope of the shock.

84.  $SLSK = TAN(SA(K-1))$

The term EKJ is used to obtain the subscript J

85.  $EKJ = NDJ + 0.60$

Begin moving along the right-running characteristics.

86. DO 131 I = L,IFIN

Calculate the J - index.

87.  $EKJ = EKJ - 0.50$

88.  $J = EKJ$

Make sure that a complete characteristics matrix is present.

89. If  $X(I,J) < 10^{-6}$ , then GO TO 132

See if I is odd or even.

90.  $MI = MOD(I,2)$

91. If  $MI = 0$ , GO TO 103

Calculate the slope of the characteristic; I is odd.

92.  $SLCH = (R(I,J) - R(I-1,J+1)) / (X(I,J) - X(I-1,J+1))$

Find the intersection of the characteristic and the shock.

93.  $XINT = (R(I-1,J+1) - RSX(K-1) + (SLSK)(XSX(K-1)) - (SLCH)(X(I-1,J+1))) / (SLSK - SLCH)$

See whether the intersection occurs between I and the I-1 points.

94. If  $X(I-1,J+1) > XINT$ , then GO TO 132

95. If  $XINT > X(I,J)$ , GO TO 131

96. If  $XINT < XSX(K-1)$ , GO TO 132

Save the characteristic point immediately upstream of the intersection.

- 97.  $XSX1(K) = X(I-1, J+1)$
- 98.  $RSX1(K) = R(I-1, J+1)$
- 99.  $NUSX1(K) = NU(I-1, J+1)$
- 100.  $TSX1(K) = T(I-1, J+1)$
- 101.  $SSX1(K) = S(I-1, J+1)$
- 102. GO TO 113

The subscript I is even. Calculate the slope of the characteristic and its intersection with the shock.

- 103.  $SLCH = (R(I, J) - R(I-1, J)) / (X(I, J) - X(I-1, J))$
- 104.  $XINT = (R(I-1, J) - RSX(K-1) + SLSK(XSX(K-1)) - (SLCH)(X(I-1, J))) / (SLSK - SLCH)$

See whether the intersection occurs between the I and the I-1 point.

- 105. If  $X(I-1, J) > XINT$ , then GO TO 132
- 106. If  $XINT > X(I, J)$ , GO TO 131
- 107. If  $XINT < XSX(K-1)$ , GO TO 132

Save the characteristic point immediately upstream of the intersection.

- 108.  $XSX1(K) = X(I-1, J)$
- 109.  $RSX1(K) = R(I-1, J)$
- 110.  $NUSX1(K) = NU(I-1, J)$
- 111.  $TSX1(K) = T(I-1, J)$
- 112.  $SSX1(K) = S(I-1, J)$

Calculate the position of the intersection.

- 113.  $XSX(K) = XINT$
- 114.  $RSX(K) = (SLCH)(XINT - X(I, J)) + R(I, J)$

Calculate the additional variables at the shock. A linear interpolation is used.

$$115. \text{RATIO} = (XINT - XSX1(K)) / (X(I,J) - XSX1(K))$$

$$116. \text{NUSX}(K) = \text{NUSX1}(K) + (\text{RATIO}) (\text{NU}(I,J) - \text{NUSX1}(K))$$

$$117. \text{TSX}(K) = \text{TSX1}(K) + (\text{RATIO}) (\text{T}(I,J) - \text{TSX1}(K))$$

$$118. \text{SSX}(K) = \text{SSX1}(K) + (\text{RATIO}) (\text{S}(I,J) - \text{SSX1}(K))$$

Calculate the position on the downstream side of the shock.

$$119. \text{XSY}(K) = \text{XSX}(K)$$

$$120. \text{RSY}(K) = \text{RSX}(K)$$

Determine the downstream streamline angle.

$$121. \text{TSY}(K) = 0.00$$

Make sure a compressive turn results.

$$122. \text{If } \text{TSX}(K) > 0, \text{ then } \text{TSY}(K) = (1.10) (\text{TSX}(K)) + (1/57.2957795)$$

Calculate the change in streamline angle.

$$123. \text{DEL} = \text{TSY}(K) - \text{TSX}(K)$$

Make sure the change in angle is not too small.

$$124. \text{If } \text{DEL} < \text{DDEL}, \text{ then } \text{TSY}(K) = \text{TSX}(K) + \text{DDEL}$$

Determine the additional properties across the shock.

$$125. \text{CALL PMTURN}$$

$$126. \text{CALL OSHOCK}$$

$$127. \text{NUSY}(K) = \text{PMANGL}(\text{MY}, \text{G})$$

$$128. \text{SSY}(K) = (\text{DSR})(\text{RG}) + \text{SSX}(K)$$

Calculate the total change in streamline angle.

$$129. \text{DELTA} = \text{TSY}(K) - \text{TSX}(K)$$

Increment the number of shock point intersections.

$$130. K = K + 1$$

131. CONTINUE

132. CONTINUE

Determine the final number of intersection points.

133.  $NSKPTS = K - 1$

There must be at least three points since a second order interpolation is used. If less than three intersections are made, a message is printed out and this subroutine returns to its calling program.

134. If  $NSKPTS < 3$ , then PRINT 106

135. If  $NSKPTS < 3$ , return

Now determine the intersection of the shock wave and the upper boundary. Set the J - subscript.

136.  $J = NDJ$

Calculate the shock angle and slope of the shock at the (K-1) point.

137.  $SA(K-1) = SWAR + TSX(K-1)$

138.  $SLSK = TAN(SA(K-1))$

Now move along the upper boundary.

139. DO 171 I = 3,NDIM1,2

Make sure a full matrix has been calculated.

140. If  $X(I,J) < 10^{-6}$ , then GO TO 172

Determine the slope of the characteristic and its intersection with the shock.

141.  $SLCH = (R(I,J) - R(I-1,J)) / (X(I,J) - X(I-1,J))$

142.  $XINT = (R(I,J) - RSX(K-1) + (SLSK)(XSX(K-1)) - (SLCH)(X(I,J))) / (SLSK - SLCH)$

See if the intersection is contained between the I and the I-1 points.

143. If  $X(I-1,J) > XINT$ , then GO TO 172

144. If  $XINT > X(I,J)$ , then GO TO 171

Save the point at the intersection.

145.  $XSX(K) = XINT$

Save the point immediately upstream of the intersection along the external jet boundary.

146.  $XSX1(K) = X(I-2,J)$

147.  $RSX1(K) = R(I-2,J)$

148.  $NUSX1(K) = NU(I-2,J)$

149.  $TSX1(K) = T(I-2,J)$

150.  $SSX1(K) = S(I-2,J)$

Determine other parameters at the intersection. A linear interpolation is used.

151.  $RSX(K) = (SLCH)(XINT - X(I,J)) + R(I,J)$

152.  $RATIO = (XINT - XSX1(K)) / (X(I,J) - XSX1(K))$

153.  $NUSX(K) = NUSX1(K) + (RATIO)(NU(I,J) - NUSX1(K))$

154.  $TSX(K) = TSX1(K) + (RATIO)(T(I,J) - TSX1(K))$

155.  $SSX(K) = SSX1(K) + (RATIO)(S(I,J) - SSX1(K))$

Determine the downstream parameters.

156.  $XSX(K) = XSX(K)$

157.  $RSX(K) = RSX(K)$

158.  $TSX(K) = 0.00$

Make sure a compressive turn occurs.

159. If  $TSX(K) > 0.00$ , then  $TSY(K) = (1.10)(TSX(K)) + (1/57.2957795)$

Calculate total change in streamline angle through the shock wave.

160.  $DEL = TSY(K) - TSX(K)$

Make sure that the change in streamline angle is not too small.

161. If  $DEL < DDEL$ , then  $TSY(K) = TSX(K) + DDEL$

Calculate remaining variables.

162. CALL PMTURN

163. CALL OSHOCK

164. NUSY(K) = PMANGL(MY,G)

165. SSY(K) = (DSR)(RG) + SSX(K)

Calculate the total streamline angle and the resulting shock wave angle.

166. DELTA = TSY(K) - TSX(K)

167. CALL SHOCK

168. SA(K) = SWAR + TSX(K)

Redefine the total number of shock points.

169. NSKPTS = K

170. GO TO 172

171. CONTINUE

Complete the matrix on the upstream point at recompression.

172. RSX1(1) = RSX(1)

173. NUSX1(1) = NUSX(1)

174. TSX1(1) = TSX(1)

175. SSX1(1) = SSX(1)

Set up the array of variables along the shock in a new list.

176. DO 187 LS = 1,NSKPTS

177. XS(LS) = XSX(LS)

178. RS(LS) = RSX(LS)

179. TS(LS) = TSX(LS)

180. NS(LS) = NUSX(LS)

181. SS(LS) = SSX(LS)

182. SAS(LS) = SA(LS)  
183. XSl(LS) = XSXl(LS)  
184. RSl(LS) = RSXl(LS)  
185. TS1(LS) = TSX1(LS)  
186. NS1(LS) = NUSX1(LS)  
187. SS1(LS) = SSX1(LS)

Reset all shock variables to zero.

188. DO 204 LS = 2,NDI  
189. XSX(LS) = 0  
190. XSY(LS) = 0  
191. RSX(LS) = 0  
192. RSY(LS) = 0  
193. TSX(LS) = 0  
194. TSY(LS) = 0  
195. NUSX(LS) = 0  
196. NUSY(LS) = 0  
197. SSX(LS) = 0  
198. SSY(LS) = 0  
199. XSX1(LS) = 0  
200. RSX1(LS) = 0  
201. NUSX1(LS) = 0  
202. TSX1(LS) = 0  
203. SSX1(LS) = 0  
204. SA(LS) = 0

Determine the sum of the linear distances along the shock wave.  
DS is the distance along the shock wave from recompression.

205. DS(1) = 0.00

206. DO 207 LS = 2,NSKPTS

207. DS(LS) = DS(LS-1) + SQRT ((XS(LS) - XS(LS-1))<sup>2</sup> + (RS(LS) - RS(LS-1))<sup>2</sup>)

Determine the total distance along the shock.

208. DTOT = DI(NSKPTS)

Determine the increment along the shock.

209. DD = DTOT/DJMI

Begin redefining the shock points. A linear interpolation is used for all variables.

210. DO 232 LS = 2,NDJ

Increment the distance

211. D = D + DD

Set the points

212. XSX(LS) = TAB(D,DS,XS,NSKPTS,1)

213. RSX(LS) = TAB(D,DS,RS,NSKPTS,1)

214. NUSX(LS) = TAB(D,DS,NS,NSKPTS,1)

215. TSX(LS) = TAB(D,DS,TS,NSKPTS,1)

216. SSX(LS) = TAB(D,DS,SS,NSKPTS,1)

217. SAS(LS) = TAB(D,DS,SAS,NSKPTS,1)

218. XSX1(LS) = TAB(D,DS,XS1,NSKPTS,1)

219. RSX1(LS) = TAB(D,DS,RS1,NSKPTS,1)

220. NUSX1(LS) = TAB(D,DS,NS1,NSKPTS,1)

221. TSX1(LS) = TAB(D,DS,TS1,NSKPTS,1)

222. SSX1(LS) = TAB(D,DS,SS1,NSKPTS,1)

223. XSY(LS) = XSX(LS)

224. RSY(LS) = RSX(LS)



Determine the Mach number upstream of the shock at that point.

225. CALL PMTURN

Calculate the shock wave angle.

226.  $SWAR = SA(LS) - TSX(LS)$

Determine the change in streamline angle across the shock.

227.  $COTDEL = TAN(SWAR) (((G + 1) (MS^2) / (2 ((MS) (SIN(SWAR)))^2 - 2)) - 1)$

228.  $DELTA = ATAN(1/COTDEL)$

Calculate the downstream streamline angle.

229.  $TSY(LS) = TSX(LS) + DELTA$

Calculate the remaining downstream properties.

230. CALL OSHOCK

231.  $NUSY(LS) = PMANGL(MY, G)$

232.  $SSY(LS) = (DSR)(RG) + SSX(LS)$

The final number of shock points is set.

233.  $NSKPTS = LS$

Set all characteristic variables to zero.

234. DO 240 I = 1, NDI

235. DO 240 J = 1, NDJ

236.  $X(I, J) = 0$

237.  $R(I, J) = 0$

238.  $NU(I, J) = 0$

239.  $T(I, J) = 0$

240.  $S(I, J) = 0$

The initial data line for this characteristics solution will be a left-running wave originating at recompression. The first estimate of the recompression shock is now read in.

241.  $EKJ = 1.60$

242. DO 249 I = 2, NSKPTS

Set the subscript J

243.  $EKJ = EKJ + 0.50$

244.  $J = EKJ$

Now read in starting line downstream of the shock.

245.  $X(I, J) = XSY(I)$

246.  $R(I, J) = RSY(I)$

247.  $NU(I, J) = NUSY(I)$

248.  $T(I, J) = TSY(I)$

249.  $S(I, J) = SSY(I)$

The initial characteristics line has now been set. Now begin the method of characteristics solution. The solution will move along a right-running characteristic until the far wake is reached. From that point, the solution proceeds along a left-running characteristic until an intersection with the shock is obtained. NINTS is the number of shock intersections. (See Fig. V-3)

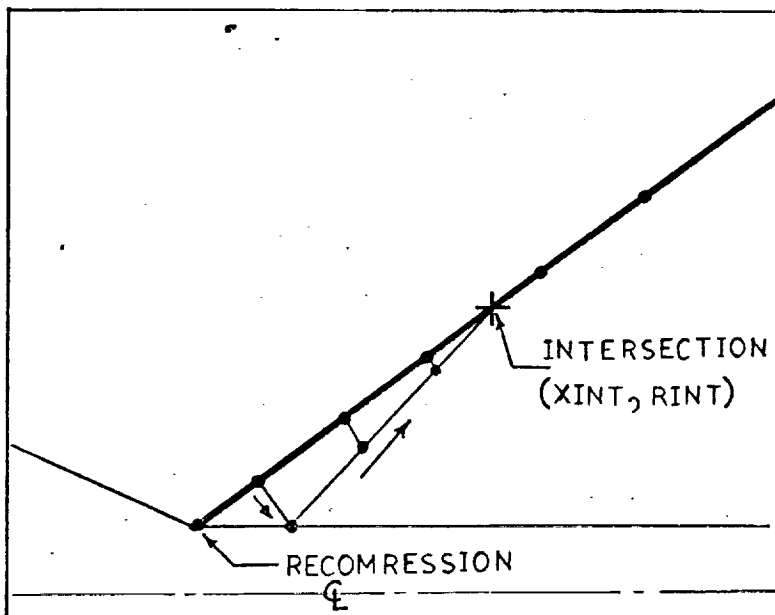


FIGURE V-3

250. NINTS = 0

Begin the characteristics solution

251. DO 388 L = 2, NSKPTS

Redefine variables. However, the old values must be saved also.

252. X2P = X2

253. R2P = R2

254. X2 = XREC

255. R2 = RSRB

Determine the far wake streamline angle.

256. TW = 0.00

Calculate the starting value of the subscript I

257. IS = (2) (L) - 1

Calculate the characteristic point on the far wake surface.

258. CALL SURF

Define a calculational integer.

259. IS1 = IS + 1

Reset old values of variables.

260. X2 = X2P

261. R2 = R2P

Now begin moving up a left-running characteristic.

262. EKJ = 1.60

263. DO 387 I = IS1, NDIM1

Set J-subscript.

264. EKJ = EKJ + 0.50

265. J = EKJ

See whether I is odd or even.

266.  $MI = MOD(I,2)$

267. If  $MI = 0$ , then GO TO 270

The subscript I is odd. Calculate the interior flow field point. If a shock wave intersection occurs, the following subroutine returns to statement number 272.

268. CALL CALC

269. GO TO 387

The subscript I is even. Again if the shock is intersected, the following subroutine returns to statement number 272.

270. CALL CALC

271. GO TO 387

A shock wave intersection has been obtained. Increment the number of intersections.

272.  $NINTS = NINTS + 1$

The location of the intersection of the characteristic wave and the shock is determined.

273. If  $MI = 0$ , then GO TO 282.

Look for an incomplete characteristic matrix: I is odd.

274. If  $|X(I-1, J+1)| < 10^{-6}$ , then GO TO 389.

Determine the subscript of the shock point immediately below the intersection.

275.  $JS = I - 2 - (NINTS - 1)$

Determine the Mach number of the characteristic point just before the intersection.

276. CALL PMTURN

Calculate the Mach angle and the total angle of the characteristics.

277.  $EMU2 = ASIN(1/EMU2)$

278.  $ANGCH = T(I-1, J) + EMU2$

Redefine the location of this point.

279.  $XQ = X(I-1, J)$

280.  $RQ = R(I-1, J)$

281. GO TO 289

The subscript I is even. Look for an incomplete characteristic matrix.

282. If  $|X(I-1, J)| < 10^{-6}$ , GO TO 389

Determine the subscript of the shock point immediately below the intersection.

283.  $JS = I - 2 - (NINTS - 1)$

Determine the Mach number of the characteristic point just before the intersection.

284. CALL PMTURN

Calculate the Mach angle and the total angle of the characteristic.

285.  $EMU2 = \text{ASIN}(1/EM2)$

286.  $ANGCH = T(I-1, J-1) + EMU2$

Redefine the location of this point.

287.  $XQ = X(I-1, J-1)$

288.  $RQ = R(I-1, J-1)$

Calculate the shock slope at the shock point just below the intersection, and the slope of the characteristic just prior to the intersection.

289.  $SLSK = \text{TAN}(SA(JS))$

290.  $SLCH = \text{TAN}(ANGCH)$

Calculate the intersection point.

291.  $XINT = (RQ - RSY(JS) + (SLSK)(XSY(JS)) - (SLCH)(XQ)) / (SLSK - SLCH)$

292.  $RINT = RSY(JS) + (SLSK)(XINT - XSY(JS))$

Determine the remaining variables downstream of the shock by an interpolation scheme.

293.  $RATIO = (XINT - XSY(JS)) / (XSY(JS+1) - XSY(JS))$   
 294.  $TINT = TSY(JS) + (RATIO) (TSY(JS + 1) - TSY(JS))$   
 295.  $NUINT = NUSY(JS) + (RATIO) (NUSY(JS+1) - NUSY(JS))$   
 296.  $SINT = SSY(JS) + (RATIO) (SSY(JS + 1) - SSY(JS))$

Determine the upstream variables at the intersection.

297.  $NUX = NUSX(JS) + (RATIO) (NUSX(JS+1) - NUSX(JS))$   
 298.  $TX = TSX(JS) + (RATIO) (TSX(JS+1) - TSX(JS))$   
 299.  $SX = SSX(JS) + (RATIO) (SSX(JS+1) - SSX(JS))$

The shock wave angle is now iterated at the intersection point. The upstream variables remain constant, but the downstream values vary with the change in shock wave angle. Make a first estimate of the downstream streamline angle and make sure that a compression occurs.

300.  $TYG = 0.00$   
 301. If  $TX > 0.00$ , then  $TYG = (1.10) (TX) + (1/57.2957795)$

Calculate the total change in streamline angle through the shock, and then calculate the Mach number upstream of the shock at the intersection.

302.  $DELTA = TYG - TX$   
 303. CALL PMTURN

Now begin the iteration procedure.

304. DO 315 NCOUNT = 1,100

Determine the shock wave angle for a given upstream Mach number and change in streamline angle.

305. CALL SHOCK

Determine the J - index.

306.  $J1 = J$   
 307. If I is an even number, then  $J = J-1$

Calculate the variables downstream of the shock at the intersection by the method of characteristics.

308. CALL SURFSK

Calculate the variables downstream of the shock at the intersection using the oblique shock relations.

309. CALL OSHOCK

Calculate the Mach number downstream of the shock which was obtained by the method of characteristics solution.

310. CALL PMTURN

Make sure that the maximum number of iterations has not been reached.

311. If NCOUNT= 100, GO TO 316

Determine the difference in the downstream Mach numbers as obtained by the two different techniques.

312.  $DFMY = MY - MINT$

Obtain a new change in streamline angle through the shock wave.

313.  $DELTA = (DELTA) (MINT/MY)$

Calculate the downstream streamline angle.

314.  $TYG = DELTA + TX$

See whether convergence has been obtained.

315. If  $|DFMY| < 0.0001$ , then GO TO 316

Redefine final downstream variables.

316.  $NUY = NUINT$

317.  $SAR = SWAR + TX$

318.  $SY = (DSR) (RG) + SX$

The remainder the shock will now be modified. The characteristics numbering system is first changed. In effect, the remainder of the flow field initial line is shifted to the next left-running wave. This is because a point was "lost" due to the intersection.

First set the starting values of the subscripts I and J.

319.  $IST = I$

320. JST = J

The variable EKJ is used to define the J-subscript.

321. EKJ = J + 1.10

322. If I is even, then EKJ = J - 0.40

Begin moving the initial line. First calculate the value of subscript J.

323. DO 333 IL = IST,NDIM1

324. EKJ = EKJ + 0.50.

325. J = EKJ

See whether IL is odd or even.

326. MI = MOD(IL,2)

Calculate another J-subscript.

327. J1 = J + 1

328. If I is even, then J1 = J

Now reset the points.

329. X(IL,J) = X(IL-1,J1)

330. R(IL,J) = R(IL-1,J1)

331. NU(IL,J) = NU(IL-1,J1)

332. T(JI,J) = T(IL-1,J1)

333. S(IL,J) = S(IL-1,J1)

Now modify the remaining portion of the shock wave. A linear interpolation (or extrapolation) is used between the two saved points along the characteristics to determine the new shock intersection point. First, reset the intersection point and determine a calculational integer. (see Fig. V-2)

334. JS1 = JS + 1

335. XSX(JS) = XINT

336. RSX(JS) = RINT



337.  $NUSX(JS) = NUX$

338.  $TSX(JS) = TX$

339.  $SSX(JS) = SX$

340.  $XSX(JS) = XSX(JS)$

341.  $RSY(JS) = RSX(JS)$

342.  $NUSY(JS) = NUY$

343.  $TSY(JS) = TY$

344.  $SSY(JS) = SY$

345.  $SA(JS) = SAR$

Now begin calculating new intersection points.

346. DO 372 K = JS1, NDI1

Calculate the slope of the shock.

347.  $SLSK = TAN(SA(K-1))$

Look for an incomplete shock array.

348. If  $XSX(K) < 10^{-6}$ , then GO TO 372.

Determine the slope of the saved two points.

349.  $SLCH = (RSX1(K) - RSX(K)) / (XSX1(K) - XSX(K))$

Fine the location of the intersection.

350.  $XINT = (RSX(K) - (RSX(K-1) + (SLSK)(XSX(K-1)) - (SLCH)XSX(K)) / (SLSK - SLCH)$

351.  $RINT = (SLCH)(XINT - XSX(K)) + RSX(K)$

Determine the ratio of distances.

352.  $RATIO = (XINT - XSX1(K)) / (XSX(K) - XSX1(K))$

Now determine the upstream variables.

353.  $XSX(K) = XINT$

354.  $RSX(K) = RINT$

355.  $NUSX(K) = NUSX1(K) + (RATIO)(NUSX(K) - NUSX1(K))$

356.  $TSX(K) = TSX1(K) + (RATIO)(TSX(K) - TSX1(K))$

357.  $SSX(K) = SSX1(K) + (RATIO)(SSX(K) - SSX1(K))$

Now determine the downstream variables.

358.  $XSX(K) = XSX(K)$

359.  $RSX(K) = RSX(K)$

Find the streamline angle.

360.  $TSY(K) = TSY(K-1)$

361. If  $TSY(K) < TSX(K)$ , then  $TSY(K) = TSX(K) + (1/57.2957795)$

Calculate the total change in streamline angle.

362.  $DEL = TSY(K) - TSX(K)$

Keep a minimum change in angle.

363. If  $DEL < DDEL$ ,  $TSY(K) = TSX(K) + DDEL$

Find the upstream Mach number at the intersection.

364. CALL PMTURN

Calculate the total change in streamline angle across the shock.

365.  $DELTA = TSX(K) - TSX(K)$

Determine the shock wave angle.

366. CALL SHOCK

Find additional variables across the shock.

367. CALL OSHOCK

Make sure an unrealistic situation has not developed.

368. If  $MY < 1.0$ , then  $MY = 1.010$

Determine the remaining downstream properties.

369.  $NUSY(K) = PMANGL(MY,G)$

370.  $SSY(K) = (DSR)(RG) + SSX(K)$

371.  $SA(K) = SWAR + TSX(K)$

372. CONTINUE

Now the downstream results are placed into the method of characteristics initial line. First see whether the starting value of I is odd or even.

373.  $MI = MOD(IST, 2)$

The term EKJ is used to determine the subscript J.

374.  $EKJ = JST + 0.10$

375. If IST is even, then  $EKJ = JST - 0.40$

Set a calculational integer.

376.  $K = JS1$

Set the downstream variables along the initial line.

377. DO 385 K1 = IST, NDIM1

Determine the J-subscript.

378.  $EKJ = EKJ + 0.50$

379.  $J = EKJ$

Set the points.

380.  $X(K1, J) = XSY(K)$

381.  $R(K1, J) = RSY(K)$

382.  $NU(K1, J) = NUSY(K)$

383.  $T(K1, J) = TSY(K)$

384.  $S(K1, J) = SSY(K)$

Increment K

385.  $K = K + 1$

Restart calculations.

386. GO TO 388

387. CONTINUE

388. CONTINUE

Skip a page.

389. PRINT 101

Print title.

390. PRINT 102

Print column headings.

391. PRINT 103

Reset number of shock points to zero.

392. NSKPTS = 0

Dimensionalize variables.

393. DO 404 K = 1,NDI

Look for a non-filled array.

394. If  $XSX(K) < 10^{-6}$ , then GO TO 404

Increment the number of shock points.

395.  $NSKPTS = NSKPTS + 1$

Dimensionalize location coordinates.

396.  $XDIM = (XSX(K)) (RPB)$

397.  $RDIM = (RSX(K)) (RPB)$

Change angles from radians to degrees.

397.  $TXD = (TSX(K)) (57.2957795)$

398.  $TYD = (TSY(K)) (57.2957795)$

Determine the upstream and downstream Mach numbers.

399. CALL PMTURN

400. CALL PMTURN

Change the shock wave angle from radians to degrees.

401.  $SWAD = (SA(K)) (57.2957795)$

Print out results.

402. PRINT 104,K,XDIM,RDIM,MINT,MYTXD,TYD,SWAD,SSX(K), SSY(K)

Punch out coordinates.

403. PUNCH 105, XDIM, RDIM

404. CONTINUE

Modify the streamline shapes due to the shock.

405. If NOSTRL = 0, GO TO 491

Define a calculational integer.

406.  $NSL1 = NOSTRL - 1$

Determine the intersection of each streamline and the shock.

407. DO 435 J = 2,NSL1

408. DO 434 I = 2,NOSPTS

Take each R- location along a streamline and see if this might intersect the shock.

409.  $XCALC = TAB(RSTR(J,I),RSX,XSX,NSKPTS,1)$

410. If  $XSTR(J,I) < XCALC$ , then GO TO 434.

An intersection has been obtained. Calculate the slope of the streamline.

411.  $SLST = TAN(TSTR(J,I))$

Determine the shock angle at the intersection.

412.  $SACALC = TAB(RSTR(J,I),RSX,SA,NSKPTS,2)$

Calculate the slope of the shock.

413.  $SLSK = TAN(SACALC)$

Determine the intersection point.

414.  $XINT = ((SLSK)(XCALC) - (SLST)(XSTR(J,I)))/(SLSK-SLST)$

415.  $RINT = (SLSK)(XINT - XCALC) + RSTR(J, I)$

Redefine the new streamline locations.

416.  $XSTR(J, I) = XINT$

417.  $RSTR(J, I) = RINT$

Use a second-order interpolation for the additional streamline variables.

418.  $TSTR(J, I) = TAB(RINT, RSX, TSX, NSKPTS, 2)$

419.  $NUINT = TAB(RINT, RSX, NUSX, NSKPTS, 2)$

Calculate the Mach number.

420. CALL PMTURN

Now set all streamline points downstream of the intersection equal to zero.

421.  $IP1 = I + 1$

422. DO 427 K = IP1, NOSPTS

Make sure the array is not exceeded.

423. If  $K > NOSPTS$ , then GO TO 435

Set variables to zero.

424.  $XSTR(J, K) = 0$

425.  $RSTR(J, K) = 0$

426.  $MSTR(J, K) = 0$

427.  $TSTR(J, K) = 0$

Set a point downstream of the shock.

428.  $XSTR(J, I + 1) = XINT + 0.250$

429.  $TSTR(J, I + 1) = TAB(RINT, RSX, TSY, NSKPTS, 2)$

430.  $NUINT = TAB(RINT, RSX, NUSY, NSKPTS, 2)$

Calculate Mach number downstream of the shock.

431. CALL PMTURN

Calculate the R- coordinate of the downstream point.

432.  $RSTR(J,I+1) = RINT + (0.25)(TAN(TSTR(J,I+1)))$

This streamline is finished.

433. GO TO 435

434. CONTINUE

435. CONTINUE

Calculate the final streamline intersection with the shock.

436. DO 437 LTP = 1,NOSPTS

437. If  $XSTR(NOSTRL,LTP) > XSX(NOSPTS)$ , and  $RSTR(NOSTRL,LTP) > 0.0001$ , then GO TO 439

An intersection is not found.

438. GO TO 491

Check to see whether the intersection is actually found.

439. If  $(RSTR(NOSTRL,LTP) - 0.250) > RSX(NSKPTS)$  then GO TO 491

An intersection has resulted.

440. IINT = LTP

Calculate the shock slope.

441.  $SLSK = TAN(SA(NSKPTS))$

Calculate the slope of the boundary.

442.  $SLB = TAN(TSTR(NOSTRL,IINT))$

Redefine the boundary point upstream of the intersection.

443.  $XB = XSTR(NOSTRL,IINT)$

444.  $RB = RSTR(NOSTRL,IINT)$

Determine the actual intersection.

445.  $XINT = (RB - RSX(NSKPTS) + (SLSK)(XSX(NSKPTS)) - (SLB)(XB)) / (SLSK - SLB)$

446.  $RINT = (SLB)(XINT - XB) + RB$

Reset the streamline point.

447.  $XSTR(NOSTRL, IINT) = XINT$

448.  $RSTR(NOSTRL, IINT) = RINT$

Determine the change in streamline angle across the shock wave.  
The cancellation of the wave is ignored for the present.

449.  $DELTA = TSY(NSKPTS) - TSX(NSKPTS)$

Redefine the upstream Mach number.

450.  $MSTR(NOSTRL, IINT) = MAMB$

Determine the shock wave angle and the variables downstream of the shock.

451. CALL SHOCK

452. CALL OSHOCK

453.  $NUSY(NSKPTS) = PMANGL(MY, G)$

Calculate the stagnation pressure downstream of the shock.

454.  $POY = (PO1)(POR)$

Calculate the required pressure ratio downstream of the shock to maintain a constant pressure boundary.

455.  $PR = POY/PA$

Determine the corresponding Mach number and P-M angle.

456.  $AMBM = SQRT(2) (PR \frac{G-1}{G} - 1) / (G-1)$

457.  $NUAMB = PMANGL(AMBM, G)$

Calculate the change in P-M angle needed to accelerate to this Mach number.

458.  $DNU = NUAMB - NUSY(NSKPTS)$

Calculate the final streamline angle at the boundary.

459.  $TSY(NSKPTS) = TSX(NSKPTS) + DNU$

See further streamline points to zero.



460.  $IP1 = I + 1$

461. DO 466 K = IP1,NOSPTS

462. If  $IP1 > NOSPTS$ , GO TO 491

463.  $XSTR(NOSTRL,K) = 0$

464.  $RSTR(NOSTRL,K) = 0$

465.  $MSTR(NOSTRL,K) = 0$

466.  $TSTR(NOSTRL,K) = 0$

Set a downstream point on this streamline.

467.  $XSTR(NOSTRL,IP1) = XINT + 0.250$

468.  $TSTR(NOSTRL,IP1) = TSY(NSKPTS)$

469.  $RSTR(NOSTRL,IP1) = RINT + (0.25) (TAN(TSTR(NOSTRL,IP1)))$

470.  $MSTR(NOSTRL,IP1) = MAMB$

See whether an additional shock point is created by the above calculations.

471. If  $/XINT - XSX(NSKPTS)/ < 0.0001$ , then GO TO 473

Add another shock point.

472.  $NSKPTS = NSKPTS + 1$

473.  $XSX(NSKPTS) = XINT$

474.  $RSX(NSKPTS) = RINT$

475.  $TSX(NSKPTS) = TSTR(NOSTRL,IINT)$

476.  $NUSX(NSKPTS) = PMANGL(MAMB,G)$

477.  $SSX(NSKPTS) = SSX(NSKPTS - 1)$

478.  $SA(NSKPTS) = SA(NSKPTS - 1)$

479.  $XSX(NSKPTS) = XSX(NSKPTS)$

480.  $RSX(NSKPTS) = RSX(NSKPTS)$

481.  $TSY(NSKPTS) = TSTR(NOSTRL,IP1)$

482. NUSY(NSKPTS) = PMANGL(AMB,G)

483. SSY(NSKPTS) = SSX(NSKPTS) + (DSR)(RG)

Dimensionalize this last shock point.

484. XDIM = (XSX(NSKPTS))(RPB)

485. RDIM = (RSX(NSKPTS))(RPB)

Change angles from radians to degrees.

486. TXD = (TSX(NSKPTS))(57.2957795)

487. TYD = (TSY(NSKPTS))(57.2957795)

488. SWAD = (SA(NSKPTS))(57.2957795)

Print out last shock point.

489. PRINT 104,NSKPTS, XDIM,RDIM,MAMB,AMBM,TXD,TYD,SWAD,SSX(NSKPTS),  
SSY(NSKPTS)

Punch out coordinates.

490. PUNCH 105, XDIM, RDIM

Reset the original stagnation pressure.

491. PO1 = POX

492. Return

VI. SUBROUTINE SOTE2B

## VI. SUBROUTINE SOTE2B

Subroutine SOTE2B is a Solution of Trancendental Equation, 2 solutions, version B. The solution is known to lie within specified limits, and the solution exists at SOL1, SOL2 = 0.

The range of  $X$  for a solution is first reduced to a  $\Delta X$  which contains the upper solution,  $XS$ , but not the lower solution ( $IND = +1$ ). To accomplish this, a search is initiated for a sign change in  $FCN$  as  $X$  is decreased from  $XU$ . The function ( $FCN = FSTEP4$ ) is shown below in Fig. VI-1. The upper solution is the one desired.

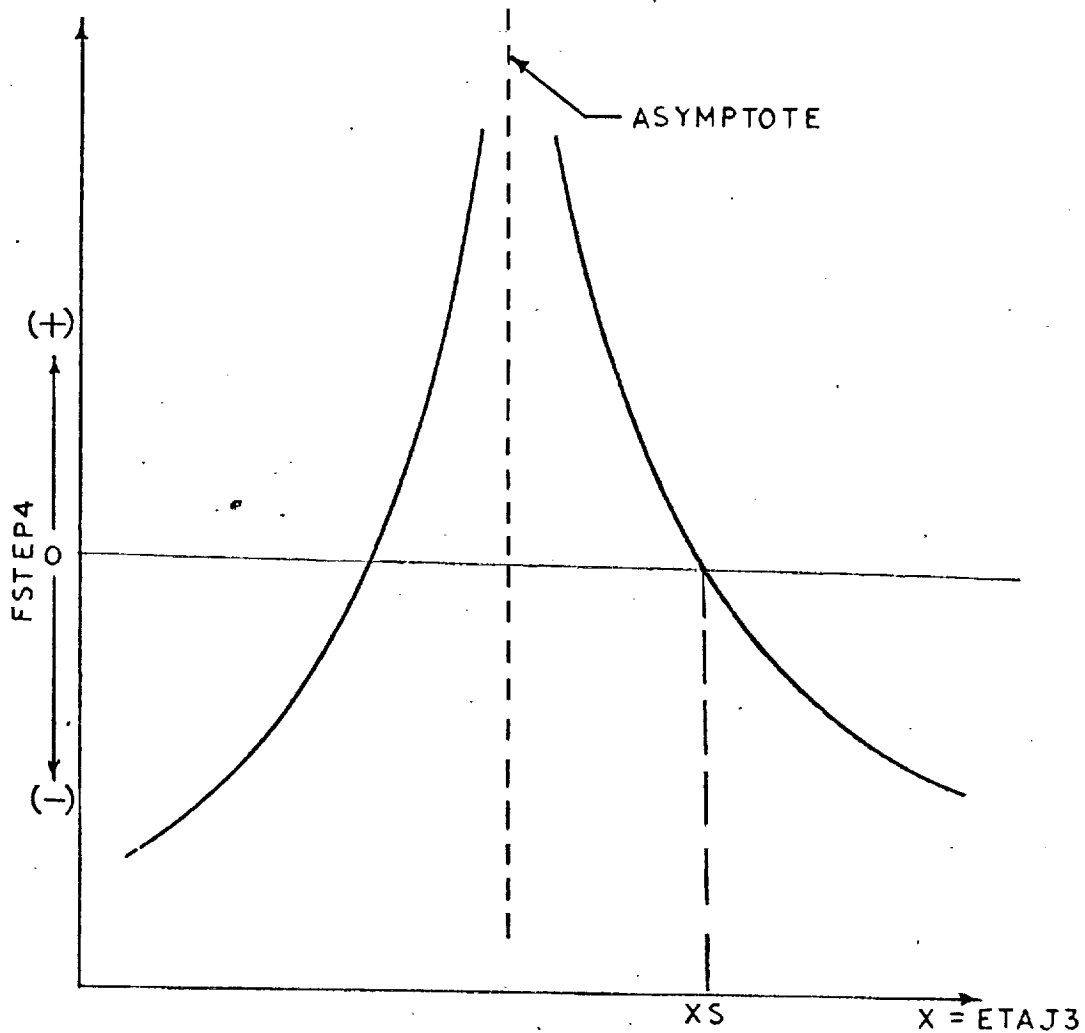


FIGURE VI-1

To save computational time, it is not advisable to use a small  $\Delta X$  at the beginning, but to continue to sweep through the interval with increasingly smaller  $\Delta X$ 's while avoiding recalculations. This is accomplished by the following scheme shown in Fig. VI-2.

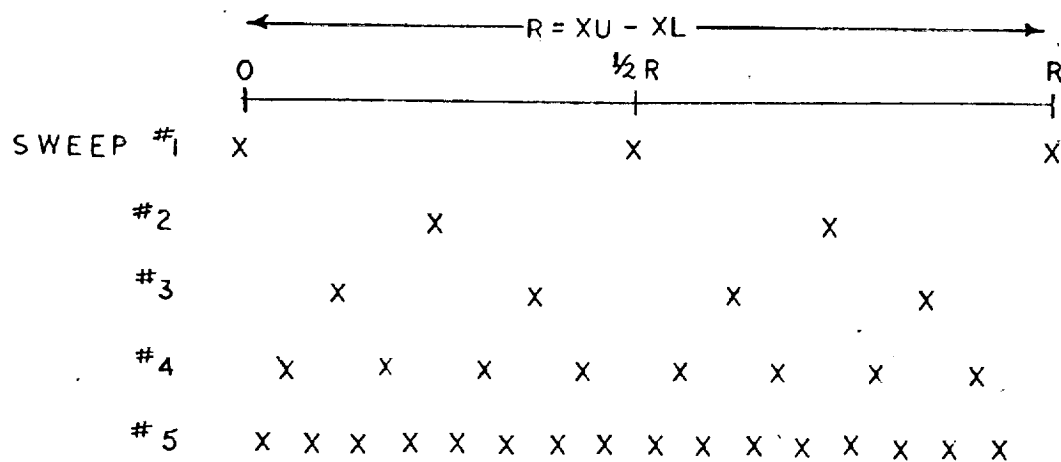


FIGURE VI-2

When a sign change is found between two values of  $X$ , the solution contained in this interval is found by interval halving sketched below in Fig. VI-3.

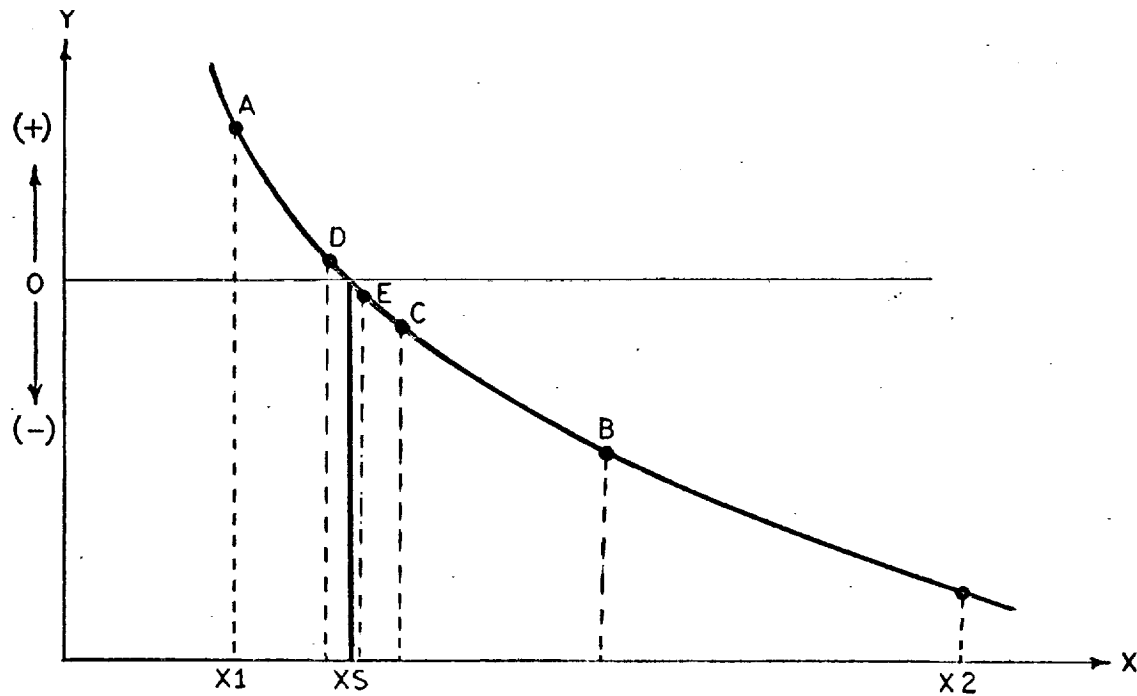


FIGURE VI-3

The calculation proceeds A-B-C-D-E until the function is between the established accuracy requirements.

#### COMMON BLOCKS

No COMMON blocks are used.

#### TPNZZL SUBROUTINES

Function subprogram FBASE6 calls SOTE2B.

Subroutine SOTE2B does not call any subroutines, but the function subprogram FSTEP4 is the functional argument of this subroutine.

## FORTTRAN SYSTEM ROUTINES

Built-in FORTRAN function ALOG is used.

### CALLING SEQUENCE

The calling sequence is:

CALL SOTE2B (FCN,XL,XU,ACX,XS,IND,\$)

FCN is the function which is to be solved

XL is the lower limit of the solution

XU is the upper limit of the solution

ACX is the accuracy requirement

XS is the final solution

IND =  $\begin{cases} +1 & \text{for the upper solution} \\ -1 & \text{for the lower solution} \end{cases}$

AB - the product of two solutions indicating whether a solution  
       has been reached  
  
 AIAC - modified accuracy requirement  
  
 CEF - fractional part of the interval at which a solution is to  
       be tried  
  
 DX - incremental part of the interval  
  
 EI - numerator of CEF  
  
 EJ - denominator of CEF  
  
 I - counter  
  
 IAC - integer of the modified accuracy requirement  
  
 ICON - counter  
  
 J - the power of 2 into which the interval is divided  
  
 K - counter  
  
 L - an integer telling in which direction the solution must proceed  
  
 N - counter  
  
 R1 - region in which the solution is contained  
  
 SOL1 - the function evaluated at the higher limit  
  
 SOL2 - the function evaluated at any point  
  
 T - the product of two solutions used as a test for convergence  
  
 X - the value of the independent variable at which the function  
       is to be evaluated  
  
 XHIGH - upper limit of the solution interval  
  
 XL2 - lower limit of the solution interval  
  
 (X) XU2 - upper limit of the solution interval  
 x



## SOLUTION METHOD

Determine the region in which the solution lies.

1.  $R1 = XU - XL$

Calculate new accuracy requirement.

2.  $AIAC = ALOG((XU-XL)/ACX)/0.69315 + 1.0$

3.  $IAC = AIAC$

Calculate the function at the upper limit.

4.  $SOL1 = FCN(XU)$

Begin an interval halving technique

5. DO 17 K = 1,12

The variable J is the number of regions.

6.  $J = 2^K$

Change J to a real number; this is the numerator of the fractional part into which the region is divided.

7.  $EJ = J$

Begin searching the region.

8. DO 16 I = 1,4100,2

The first subdivision is a special case.

9. If K = 1, and I = 3, then N = 2

Make sure extra points are not calculated, i.e., those outside the region.

10. If N > J, then GO TO 17

Determine the numerator of the fractional part of the region.

11.  $EI = I$

Set a counter.

12.  $ICON = K$

Calculate the fraction.

13.  $CEF = EI/EJ$

Calculate the value of X at which the function is to be evaluated.  
(See Fig. XI-2).

14.  $X = XU - (R1)(CEF)$

Move out of DO-loop to perform additional calculations.

15. GO TO 20

16. CONTINUE

17. CONTINUE

No solution is found. Print out message and return to the statement number described in the 7th argument.

18. PRINT 101

19. RETURN 7

Determine the value of the function at X.

20.  $SOL2 = FCN(X)$

Look for a change in sign of the functions.

21.  $T = (SOL1)(SOL2)$

22. If  $T \geq 0$ , then GO TO 16

Only one solution is found if on the first try a sign change is indicated.

23. If  $N = 2$  and  $J = 2$ , GO TO 50

See whether the upper or lower solution is desired.

24. If  $IND > 0$ , GO TO 28

The lower solution is wanted. Redefine variables.

25.  $XHIGH = X$

26.  $XL2 = X - R1/EJ$

27. TO TO 30

The upper solution is desired.

28.  $XHIGH = X + R1/EJ$

29.  $XL2 = X$

Determine upper limit on X.

30.  $XU2 = XHIGH$

Now begin interval halving procedure. (See Fig. VI-3) Redefine the X- value.

31.  $X = XL2$

Determine the increment.

32.  $DX = (XU2 - XL2)/2$

Recalculate the function.

33.  $SOL1 = FCN(X)$

Set the direction integer.

34.  $L = + 1$

Calculate a new value of X.

35.  $X = X + (L)(DX)$

Increment counter.

36.  $ICON = ICON + 1$

See if a solution is reached.

37. If  $ICON > IAC$ , GO TO 45.

A solution is not found yet. Recalculate new upper function.

38.  $SOL2 = FCN(X)$

Print out the value of X and the function.

39. PRINT 100,X,SOL2

Test for sign change.

40.  $T = (SOL1)(SOL2)$

41. If  $T > 0$ , then GO TO 43.

A sign change is indicated. Change direction.

42.  $L = -L$

Redefine function.

43.  $SOL1 = SOL2$

Cut interval in half.

43.  $DX = DX/2$

Repeat calculation procedure.

44. GO TO 28

A solution is reached.

45.  $XS = X$

Make a final check.

46.  $AB = (FCN(XS+ACX) (FCN(XS-ACX)))$

47. If  $AB < 0$ , return

Problems have developed such that the required accuracy has not been maintained. A statement is printed out, and the solution obtained is returned.

48. PRINT 102, XS, SOL1

49. Return

Only one solution is indicated. A statement to this effect is printed and the solution continues.

50. PRINT 103

51. GO TO 24

## VII. SUBROUTINE SOTE

## VII. SUBROUTINE SOTE

Subroutine SOTE obtains a Solution of a Trancendental Equation with only one root. The solution is at SOL1, SOL2 = 0. An interval halving technique is used just like the one described in Subroutine SOTE2B.

### COMMON BLOCKS

No COMMON blocks are used.

### TPNZZL SUBROUTINES

Subroutine FBASE6 calls SOTE.

SOTE calls no subroutines, but the function subprogram F2D is used as an argument of SOTE.

### FORTRAN SYSTEM ROUTINES

No built-in FORTRAN functions are used.

### CALLING SEQUENCE

The calling sequence is:

CALL SOTE (FCN,XL,XU,ACFX,XS,\$)

FCN is the function which is to be solved.

XL is the lower limit of the solution.

XU is the upper limit of the solution.

ACFX is the accuracy of the solution.

XS is the value of the solution.

- DX     - incremental part of the interval
- ISAFY - an integer counting the number of sweeps
- L       - an integer telling in which direction the solution must proceed
- SOL1   - the function evaluated at the higher limit
- SOL2   - the function evaluated at another point
- T       - the product of two solutions used as a test for convergence
- X       - the value of the independent variable at which the function  
          is to be evaluated

## SOLUTION METHOD

Define a counter.

1.  $ISAF\bar{T}Y = 1$

Determine starting value of X.

2.  $X = XL$

Calculate the increment.

3.  $DX = (XU - XL)/2$

Calculate the function.

4.  $SOL1 = FCN(X)$

See if a solution has been reached.

5. If  $|SOL1| < ACFX$ , GO TO 17

Define direction integer.

6.  $L = + 1$

Recalculate X

7.  $X = X + (L)(DX)$

Increment counter.

8.  $ISAF\bar{T}Y = ISAF\bar{T}Y + 1$

See if maximum number of calculations has been reached.

9. If  $ISAF\bar{T}Y > 30$ , THEN GO TO 19

Recalculate function at the upper limit.

10.  $SOL2 = FCN(X)$

See if convergence has been reached.

11. If  $|SOL2| < ACFX$ , then GO TO 17

Test for sign change.

12. If  $T > 0$ , GO TO 14



A sign change is indicated. Reverse direction.

13.  $L = -L$

Redefine function.

14.  $SOL1 = SOL2$

The increment is now halved.

15.  $DX = DX/2$

Redo calculation procedure.

16. GO TO 7

A solution has been obtained.

17.  $XS = X$

18. Return

No solution was obtained. Print out message and return to calling program.

19. PRINT 22,X

20. RETURN 6

## VIII. SUBROUTINE SETUP

## VIII. SUBROUTINE SETUP

Subroutine SETUP resets the initial non-characteristic line when the characteristic matrix has been completed. The initial line may be transposed from the last line, or maybe established at equal increments along this line.

### COMMON BLOCKS

COMMON blocks PARAM, PLCBLK, and SIZE are used.

### TPNZZL SUBROUTINES

The subroutine SETUP is called by the subroutine FLOW.

Subroutine SETUP calls subroutines and functions STRLNE and TAB.

### FORTRAN SYSTEM ROUTINES

Built-in FORTRAN function SQRT is used.

### CALLING SEQUENCE

The calling sequence is:

CALL SETUP (NDF,ILOC,\$,IOPTSP)

NDF is the final value of the axial subscript in the characteristics solution

ILOC is an integer which locates the position of the characteristics solution in the nozzle.

D - non-dimensional distance along final line  
 DINC - incremental distance along new starting line (non-dimensional)  
 D1 - non-dimensional distance along new starting line  
 I - Subscript  
 ILOCN - integer telling which portion of the flow field is being calculated (See Fig. IV-1)  
 J - Subscript  
 LOCN - integer telling which portion of the flow field is being calculated (See Fig. I-3)  
 NDI - maximum number of I-subscripts in the characteristic matrix  
 NDJ - maximum number of J-subscripts in the characteristic matrix  
 NDJML - NDJ-1  
 NU - Prandtl-Meyer angle (radians) at each point in the characteristic matrix  
 NU1 - array of Prandtl-Meyer angles (radians) at each point on the final line  
 R - non-dimensional radial coordinate at each point in the characteristic matrix  
 R1 - array of non-dimensional radial coordinates at each point on the final line  
 S - entropy at each point in the characteristic matrix  
 S1 - array of entropies at each point on the final line  
 T - streamline angle (radians) at each point in the characteristic matrix  
 T1 - array of streamline angles (radians) at each point on the final line  
 X - non-dimensional axial coordinate at each point in the characteristic matrix  
 X1 - array of non-dimensional axial coordinates at each point on the final line

## SOLUTION METHOD

Calculate streamlines in completed characteristic matrix.

1. CALL STRLNE

Define calculational integer.

2. NDJML = NDJ - 1

Redefine last non-characteristic line.

3. DO 8 J = 1,NDJ

4. X1(J) = X(NDF,J)

5. R1(J) = R(NDF,J)

6. NU1(J) = NU(NDF,J)

7. T1(J) = T(NDF,J)

8. S1(J) = S(NDF,J)

Calculate distances along line.

9. D(1) = 0.00

10. DO 11 J = 2,NDJ

11.  $D(J) = (D(J-1) + \text{SQRT}((X1(J) - X1(J-1))^2 + (R1(J) - R1(J-1))^2))$

Reset characteristic matrix to zero.

12. DO 18 I = 1,NDI

13. DO 18 J = 1,NDJ

14. X(I,J) = 0

15. R(I,J) = 0

16. NU(I,J) = 0

17. T(I,J) = 0

18. S(I,J) = 0

Set up an even increment spacing.

19.  $DINC = D(NDJ)/NDJMI$

Set first point.

20.  $D1 = 0.00$

21.  $X(1,1) = X1(1)$

22.  $R(1,1) = R1(1)$

23.  $NU(1,1) = NU1(1)$

24.  $T(1,1) = T1(1)$

25.  $S(1,1) = S1(1)$

If the characteristics solution has just begun, a different procedure is used.

26. If  $LOCN = 0$ , then GO TO 36

Set up an even spacing.

27. DO 33 J = 2,NDJ

Increment the distance.

28.  $D1 = D1 + DINC$

Reset points using a linear interpolation.

29.  $X(1,J) = TAB(D1,D,X1,NDJ,1)$

30.  $R(1,J) = TAB(D1,D,R1,NDJ,1)$

31.  $NU(1,J) = TAB(D1,D,NU1,NDJ,1)$

32.  $T(1,J) = TAB(D1,D,T1,NDJ,1)$

33.  $S(1,J) = TAB(D1,D,S1,NDJ,1)$

If recompression has been reached make a normal return; otherwise a special return is used.

34. If  $LOC = 3$ , return

35. RETURN 3

A simple resubstitution of points is used.

36. DO 41 J = 2,NDJ

37. X(1,J) = X1(J)

38. R(1,J) = R1(J)

39. NU(1,J) = NU1(J)

40. T(1,J) = T1(J)

41. S(1,J) = S1(J)

Return to calling program.

42. RETURN

## IX. SUBROUTINE LINEAR



## IX. SUBROUTINE LINEAR

Subroutine LINEAR obtains a solution at zero for a single-valued function. This is done by a series of linear interpolations on extrapolations from two calculated points. Fig. IX-1 demonstrates the method of solution. Points A and B are evaluated and the function is projected to a solution at Point C. Points C and B then combine to predict a solution at Point D. Finally, Points C and D are used to calculate the point E which is within the desired accuracy.

Three arguments in the FCN are required.

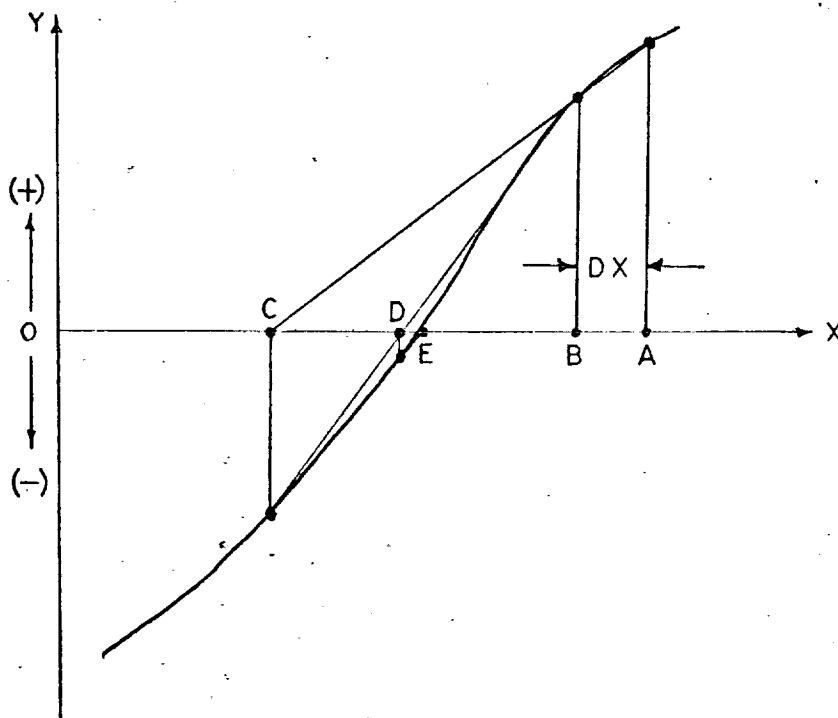


FIGURE IX-1

### COMMON BLOCKS

No COMMON blocks are used.

### TPNZL SUBROUTINES

Subroutine BASE6 calls LINEAR

LINEAR does not call any subroutines, but function subprogram FBASE6 is a functional argument in LINEAR.

## FORTRAN SYSTEM ROUTINES

No built-in FORTRAN functions are used.

### CALLING SEQUENCE

The calling sequence is:

CALL LINEAR (FCN,XEST,ACX,XS,T,N,DXLIN,\$,NEPS)

FCN is the function to be solved.

XEST is the first estimate of the solution.

ACX is the accuracy desired.

XS is the value of the solution.

T is the value of the test for convergence.

N is the number of iterations.

DXLIN is the first increment (DXLIN < XEST).

NEPS =  $\begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$

- DX - the incremental starting distance on the ordinate (See Fig. IX-1)
- S - slope of the line drawn between two calculated points
- X1 - the value of the ordinate of one of the points
- X2 - the value of the ordinate of the other point
- X3 - the value of the ordinate at the projected solution
- Y1 - the value of FCN at X1
- Y2 - the value of FCN at X2
- Y3 - the value of FCN at X3

## SOLUTION METHOD

Redefine the increment.

1.  $DX = DXLIN$

Determine the first estimate of the solution.

2.  $X1 = XEST$

Set counter.

3.  $N = 1$

Evaluate the function at  $X1$

4.  $Y1 = FCN(X1, \$27, NEPS)$

See whether a solution has been reached.

5.  $T = /Y1/ - ACX$

6. If  $T < 0$ ,  $X3 = X1$

7. If  $T < 0$ , GO TO 25

A solution was not reached. Determine a new value of  $X$ .

8.  $X2 = X1 - DX$

Evaluate the function at  $X2$ .

9.  $Y2 = FCN(X2, \$27, NEPS)$

See whether a solution has been reached.

10.  $T = /Y2/ - ACX$

11. If  $T < 0$ ,  $X3 = X2$

12. If  $T < 0$ , then GO TO 25

No solution was obtained. Calculate the slope of the line drawn between the two points.

13.  $S = (Y1 - Y2) / (X1 - X2)$

Extrapolate or interpolate to a solution at  $Y = 0$ . (See Fig. IX-1)

14.  $X3 = - (Y2/S) + X2$

Evaluate the function at this point.

15.  $X3 = FCN(X3, \$27, NEPS)$

See whether a solution has been reached.

16.  $T = /Y3/ = ACX$

17. If  $T < 0$ , then GO TO 25.

See if the maximum number of iterations has been exceeded.

18. If  $N > 15$ , then GO TO 27.

Reincrement counter.

19.  $N = N + 1$

Update variables.

20.  $X1 = X2$

21.  $Y1 = Y2$

22.  $X2 = X3$

23.  $Y2 = Y3$

Search for new solution.

24. GO TO 13

A solution has been reached.

25.  $XS = X3$

26. RETURN

No solution was obtained. The last value of X is returned, however.

27.  $XS = X3$

Print out message and make a non-standard return.

38. PRINT 101

29. PRING 102, XS, ACX, T

30. RETURN 8

X. SUBROUTINE LNEAR1

## X. SUBROUTINE LNEAR1

Subroutine LNEAR1 is basically the same as subroutine LINEAR, except that the function, FCN, has only one argument. A linear interpolation or extrapolation from two calculated points is used to determine a solution at  $Y = 0$ . (See Fig. IX-1.) Again, the function must be single-valued.

### COMMON BLOCKS

No COMMON blocks are used.

### TPNZZL SUBROUTINES

Function subprogram FBASE6 calls LNEAR1.

LNEAR1 does not call any subroutines, but function subprogram ERFD3 is a functional argument in LNEAR1.

### FORTRAN SYSTEM ROUTINES

No built-in FORTRAN functions are used.

### CALLING SEQUENCE

The calling sequence is:

CALL LNEAR1 (FCN,XEST,ACX,XS,T,N,DXLIN,\$)

FCN is the function to be evaluated.

XEST is the first estimate of the solution.

ACX is the accuracy desired.

T is the value of the test for convergence.

N is the number of iterations.

DXLIN is the first increment ( $DXLIN < XEST$ ).

- DX     - the incremental starting distance on the ordinate (See Fig. IX-1)
- S       - the slope of the line drawn between two calculated points
- X1     - the value of the ordinate at one of the points
- X2     - the value of the ordinate at the other point
- X3     - the value of the ordinate at the projected solution
- Y1     - the value of FCN at X1
- Y2     - the value of FCN at X2
- Y3     - the value of FCN at X3



# SOLUTION METHOD

Redefine the increment

1.  $DX = DXLIN$

Determine the first estimate of the solution.

2.  $X1 = XEST$

Set counter.

3.  $N = 1$

Evaluate the function at  $X1$ .

4.  $Y1 = FCN(X1)$

Determine a new value of  $X$ .

5.  $X2 = X1 - DX$

Evaluate the function at  $X2$ .

6.  $Y2 = FCN(X2)$

Calculate the slope of the line drawn between the two points.

7.  $S = (Y1 - Y2) / (X1 - X2)$

Extrapolate or interpolate to a solution at  $Y = 0$ .

8.  $X3 = - (Y2/S) + X2$

Evaluate the function at  $X3$ .

9.  $Y3 = FCN(X3)$

See whether a solution has been obtained.

10.  $T = /Y3/ - ACX$

11. If  $T < 0$ , then GO TO 19

See if the maximum number of iterations has been exceeded.

12. If  $N > 15$ , then GO TO 21

Increment counter.

13.  $N = N + 1$

Redefine points.

14.  $X1 = X2$

15.  $Y1 = Y2$

16.  $X2 = X3$

17.  $Y2 = Y3$

Search for a new solution.

18. GO TO 7

A solution is reached.

19.  $XS = X3$

20. RETURN

No solution was found in 15 iterations. The last value is returned by a non-standard return. A message is printed out.

21.  $XS = X3$

22. PRINT 101

23. PRINT 102, XS, ACX, T

24. RETURN 8

## XI. SUBROUTINE HYPER

## XI. SUBROUTINE HYPER

Subroutine HYPER finds a solution at  $Y = -1.0$  of a function whose overall shape is that of a hyperbola with an asymptote along  $Y = +1.0$ . This type of base pressure solution curve results when base bleed is present. The technique used is basically the same as that used in LINEAR, except that a hyperbolic extrapolation or interpolation is used instead of a linear extrapolation or interpolation. Point A is calculated at the estimated solution and Point B is calculated at a value  $DX$  away from the estimated solution. Points A and B then project the solution to Point C. Points B and C project the solution to Point D, and finally, Points C and D determine the solution at Point E which is within the accuracy requirement.

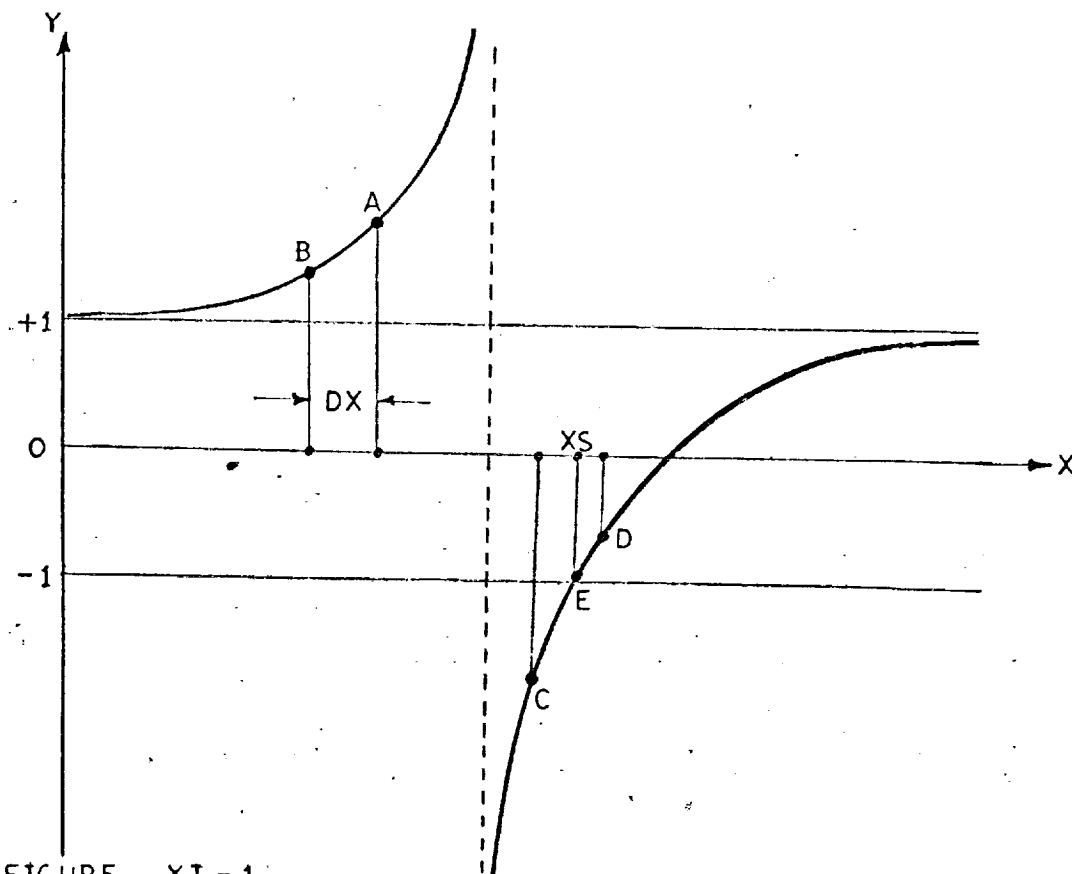


FIGURE XI-1

### COMMON BLOCKS

No COMMON blocks are used.

## TPNZZL SUBROUTINES

Subroutine BASE6 calls HYPER.

HYPER does not call any subroutines, although the function subprogram FBASE6 is the functional argument in HYPER.

## FORTRAN SYSTEM ROUTINES

No built-in FORTRAN functions are used.

## CALLING SEQUENCE

The calling sequence is:

CALL HYPER (FCN,XEST,ACX,XS,DXLIN,\$,NEPS)

FCN is the function to be evaluated

XEST is the first estimate of the solution

ACX is the accuracy requirement desired

XS is the value of the solution

DXLIN is the first increment (DXLIN < XEST)

$$NEPS = \begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$$

- A - coefficient in the hyperbolic equation
- B - coefficient in the hyperbolic equation
- DX - the incremental starting distance on the ordinate (See Fig. XI-1)
- N - number of iterations
- T - test for a solution
- X1 - the value of the ordinate at one of the points
- X2 - the value of the ordinate at another point
- X3 - the value of the ordinate at the projected solution
- Y1 - the value of FCN at X1
- Y2 - the value of FCN at X2
- Y3 - the value of FCN at X3

## SOLUTION METHOD

Set counter.

1.  $N = 1$

Redefine the increment.

2.  $DX = DXLIN$

Determine the initial estimate of X.

3.  $X1 = XEST$

Evaluate the function at X1.

4.  $Y1 = FCN(X1, \$31, NEPS)$

See whether a solution has been reached.

5.  $T = /Y1 + 1.0/ - ACX$

6. If  $T < 0$ , then  $X3 = X1$

7. If  $T < 0$ , GO TO 29

No solution at X1. Calculate a new value of X.

8.  $X2 = X1 - DX$

Evaluate the function at X2.

9.  $Y2 = FCN(X2, \$31, NEPS)$

See whether a solution has been obtained at X2.

10.  $T = /Y2 + 1.0/ - ACX$

11. If  $T < 0$ , then  $X3 = X2$

12. If  $T < 0$ , GO TO 29

Two "constants" in the hyperbolic equation are now evaluated.

13.  $B = ((Y1-1)(X1) - (Y2-1)(X2))/(Y1-Y2)$

14.  $A = 1/((Y1-1)(X1-B))$

Extrapolate or interpolate with a hyperbolic curve to solution at  $Y = -1.0$ . (See Fig. XI-1)

15.  $X3 = B - 0.50/A$

Make sure unrealistic values do not result.

16. If  $X3 < 0.0010$ , then  $X3 = 0.010 + (0.010)(N)$

17. If  $X3 > 2.50$ , and  $N < 5$ , then  $X3 = 2.5 - (0.10)(N)$

18. If  $X3 > 4.00$ , then  $X3 = 4.0 + (0.10)(N)$

Evaluate the function at  $X3$ .

19.  $Y3 = FCN(X3, \$31, NEPS)$

See whether a solution is reached.

20.  $T = /Y3 + 1.0/ - ACX$

21. If  $T < 0$ , GO TO 29

See whether the maximum number of iterations has been exceeded.

22. If  $N > 15$ , GO TO 31

Increment the counter.

23.  $N = N + 1$

Reset points.

24.  $X1 = X2$

25.  $Y1 = Y2$

26.  $X2 = X3$

27.  $Y2 = Y3$

Restart solution procedure.

28. GO TO 13

A solution is obtained.

29.  $XS = X3$

30. Return

No solution was reached.

31.  $XS = X3$



Print out message and results.

32. PRINT 101

33. PRINT 102, XS, ACX, T

Make a non-standard return.

34. Return 8

XII. SUBROUTINE SURFSK

## XII. SUBROUTINE SURFSK

Subroutine SURFSK performs the method of characteristics solution at the intersection of a shock wave and a left-running characteristic (See Fig. XII-1). Only the downstream results are obtained from this subroutine.

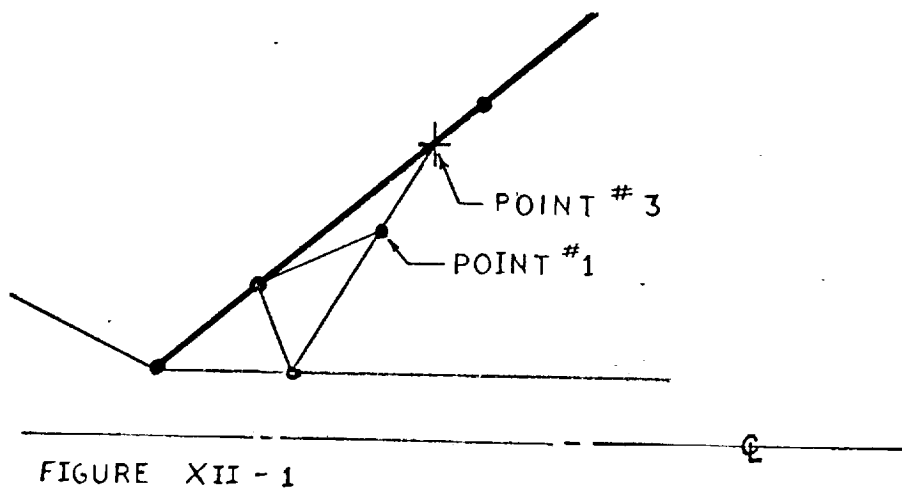


FIGURE XII - 1

### COMMON BLOCKS

No COMMON blocks are required.

### TPNZZL SUBROUTINES

Subroutine SSHAPE calls SURFSK for the recompression shock. When a lip shock is present, SURFSK is called by subroutine LIPSHK.

SURFSK calls the subroutine PMTURN.

### FORTRAN SYSTEM ROUTINES

Built-in FORTRAN functions ASIN, COS, SIN, and SQRT are used.

## CALLING SEQUENCE

The calling sequence is:

CALL SURFSK (\$,NU3,T3,S3,X3,R3,NU1,T1,S1,X1,R1,T2,S2,G,RG,NEPS)

NU3 is the value of the Prandtl-Meyer turn angle on the downstream side of the shock (radians).

T3 is streamline angle (radians) on the downstream side of the shock.

S3 is the entropy on the downstream side of the shock.

X3,R3 are the axial and radial location of the shock.

NU1 is the Prandtl-Meyer angle (in radians) at point #1 whose left-running characteristic intersects the shock.

T1 is the streamline angle (radians) at the point whose characteristic intersects the shock.

S1 is the entropy at the point whose characteristic intersects the shock.

X1,R1 are the axial and radial locations of the point whose characteristic intersects the shock.

T2 is the value of the downstream streamline angle (radians) at the point where the characteristic wave intersects the shock.

S2 is the value of the downstream entropy at the point where the characteristic wave intersects the shock.

G is the ratio of specific heats.

RG is the gas constant of the nozzle gases.

NEPS =  $\begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$

C1     - the value of the integrand (Equation I-34) at point #1  
 C13    - the average value of the integrand (Equation I-34) between  
           points #1 and #3  
 C3     - the value of the integrand (Equation I-34) at point #3  
 EPS    - real number (=1.0 for axisymmetric flow; =0.0 for planar flow)  
 F1I    - the isentropic portion of Equation I-37a  
 IDUM    - an integer counter used in iteration  
 K1     - the value of the rotational effects (Equation I-35) at point #1  
 K13    - the average value of the rotational effects (Equation I-35)  
           between points #1 and #3  
 K3     - the value of the rotational effects (Equation I-35) at point #3  
 MU1    - the Mach angle (radians) at point #1  
 MU3    - the Mach angle (radians) at point #3  
 MU3G   - the initial estimate of the Mach angle (radians) at point #3  
 M1     - the Mach number at point #1  
 M3     - the Mach number at point #3  
 M3G    - the initial estimate of the Mach number at point #3  
 NLOOPS - the number of iterations in the characteristics solution  
 NU3G   - the first estimate of the Prandtl-Meyer angle (radians) at  
           point #3  
 R13    - the average non-dimensional radius between points #1 and #3  
 Z13    - the non-dimensional distance between points #1 and #3

## SOLUTION METHOD

The term NEPS is equal to unity for axisymmetric flow and is zero for planar configurations. Change this variable to a real number.

1.  $EPS = NEPS$

Assign the number of iterations to be made.

2.  $NLOOPS = 5$

Set the streamline angle.

3.  $T3 = T2$

Determine the Mach number of the point before the intersection and the corresponding Mach angle.

4.  $CALL\ PMTURN$

5.  $MU1 = ASIN(1/M1)$

Set the entropy.

6.  $S3 = S2$

Check for errors.

7. If  $R3 < 0$ , then  $RETURN\ 1$

Calculate the average radius between the intersection and the characteristic point just before the intersection.

8.  $R13 = (R1 + R3)/2$

Determine the distance along the characteristic.

9.  $Z13 = SQRT((R1 - R3)^2 + (X1 - X3)^2)$

Calculate the value of the integrand. This takes into account axisymmetric effects. (Equation I-34)

10.  $C1 = ((SIN(MU1))(SIN(T1)))/R13(EPS)$

Take into account the rotational aspect of the flow. (Equation I-35)

11.  $K1 = (SIN(MU1))(COS(MU1))/G$

Obtain a first estimate of the Prandtl-Meyer(P-M) angle at the intersection downstream of the shock.

$$12. \quad NU3G = NU1 - T1 + T3 + (C1)(Z13) - (K1)(S3-S1)/RG$$

Determine the corresponding Mach number and the Mach angle.

13. CALL PMTURN

$$14. \quad MU3G = ASIN(1/M3G)$$

Calculate the integrand and the rotational effects at the intersection. (Equations I-34 and I-35)

$$15. \quad C3 = ((SIN(MU3G))(SIN(T3))/R3)(NEPS)$$

$$16. \quad K3 = (SIN(MU3G))(COS(MU3G))/G$$

Iterate for a solution.

17. DO 25 IDUM = 1,NLOOPS

Take average values of the integrand and the rotational term.

$$18. \quad C13 = ((C1)(R13/R1) + C3)/2$$

$$19. \quad K13 = (K1 + K3)/2$$

Calculate the isentropic P-M angle at the point.

$$20. \quad F1I = NU1 - (T1-T3) + (C13)(Z13)$$

Include rotational effects.

$$21. \quad NU3 = F1I - (K13)(S3-S1)/RG$$

Determine the corresponding Mach number and Mach angle.

22. CALL PMTURN

$$23. \quad MU3 = ASIN(1/M3)$$

Recalculate the modified values of the integrand and the rotational term. (Equations I-34 and I-35)

$$24. \quad C3 = ((SIN(MU3))(SIN(T3))/R3)(EPS)$$

$$25. \quad K3 = (SIN(MU3))(COS(MU3))/G$$

26. RETURN

Errors have resulted in Mach number calculations. The results are printed out.

27. PRINT 101, NU1
28. RETURN
29. PRINT 102, NU3G
30. RETURN
31. PRINT 103, NU3
32. RETURN



XIII. SUBROUTINE SURFP

### XIII. SUBROUTINE SURFP

Subroutine SURFP calculates the characteristic point on a solid lower boundary, i.e., the plug surface.

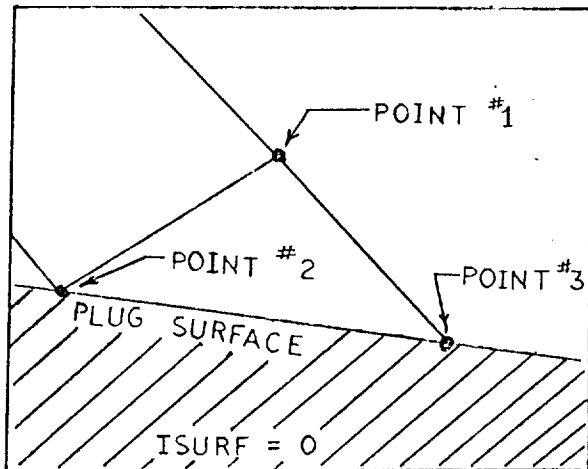


FIGURE XIII-1

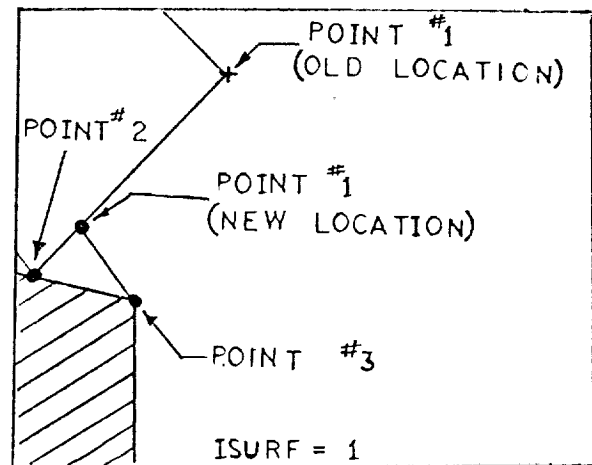


FIGURE XIII-2

#### COMMON BLOCKS

COMMON blocks PTNOS and TPN are used.

#### TPNZL SUBROUTINES

Subroutine FLOW calls SURFP.

SURFP calls subroutine PMTURN and function TAB.

#### FORTTRAN SYSTEM ROUTINES

Built-in functions ASIN, COS, SIN, SQRT, and TAN are used.

## CALLING SEQUENCE

The calling sequence is:

CALL SURFP (\$,NU3,T3,S3,X3,R3,NU1,T1,S1,X1,R1,NU2,T2,S2,X2,R2,G,  
RG,NEPS,ISURF)

NU3 is the Prandtl-Meyer turn angle on the plug surface.

T3 is the streamline angle on the plug surface.

S3 is the entropy at the point on the plug surface.

X3,R3 are the axial and radial location of the point on the plug surface.

NU1 is the Prandtl-Meyer angle of the point whose right-running characteristic intersects the plug.

T1 is the streamline angle at this point.

S1 is the entropy at this point.

X1,R1 are the axial and radial coordinates of this point.

NU2 is the Prandtl-Meyer angle of the first upstream plug point.

T2 is the streamline angle at this point.

S2 is the entropy at this point.

X2,R2 are the axial and radial coordinates of this point.

G is the ratio of specific heats.

RG is the gas constant.

NEPS =  $\begin{cases} 1 & \text{for axisymmetric flow.} \\ 0 & \text{for planar flow.} \end{cases}$

ISURF =  $\begin{cases} 0 & \text{when the plug base has NOT been reached.} \\ 1 & \text{when the plug base has been reached.} \end{cases}$

C1 - the value of the integrand (Equation I-34) at point #1  
 C13 - the average value of the integrand (Equation I-34) between points #1 and #3  
 C3 - the value of the integrand (Equation I-34) at point #3  
 EPS - real number (=1.0 for axisymmetric flow; =0.0 for planar flow)  
 F1I - the isentropic portion of Equation I-37a  
 IDUM - an integer counter used in iteration  
 IP - a subscript of the array of the plug points  
 K1 - the value of the rotational effects (Equation I-35) at point #1  
 K13 - the average value of the rotational effects (Equation I-35) between points #1 and #3  
 K3 - the value of the rotational effects (Equation I-35) at point #3  
 MU1 - the Mach angle (radians) at point #1  
 MU3 - the Mach angle (radians) at point #3  
 MU3G - the initial estimate of the Mach angle (radians) at point #3  
 M1 - the Mach number at point #1  
 M3 - the Mach number at point #3  
 M3G - the initial estimate of the Mach number at point #3  
 NLOOPS - the number of iterations in the characteristics solution  
 NOIPTS - the number of initial data line points  
 NOPPTS - the number of plug coordinate points  
 NUINT - the value of the Prandtl-Meyer angle (radians) at the new location of point #1  
 NU3G - the initial estimate of the Prandtl-Meyer angle (radians) at point #3  
 PMACH - the Mach number corresponding to NUINT  
 PMU - the Mach angle (radians) corresponding to NUINT

RAP - the non-dimensional plug radius at the point just upstream of the plug base  
 RATIO - a ratio of axial distances at the plug base  
 RCP - the non-dimensional radius at point #1  
 RINT - the non-dimensional radius at the new location of point #1  
 RP - an array of non-dimensional numbers of the radial coordinates of the plug  
 RPB - the plug base radius (inches)  
 R13 - the average non-dimensional radius between points #1 and #3  
 R2A - the non-dimensional plug radius of the point just downstream of the characteristic intersection  
 SINT - the entropy at the new location of point #1  
 SLAC - an approximate value of the slope of the plug contour at the characteristic intersection  
 SLCH - the slope of the characteristic from points #1 to #3  
 SL1 - the slope of the right-running characteristic from point #1  
 SL2 - the slope of the plug contour at point #3  
 TINT - the streamline angle (radians) at the new location of point #1  
 TP - the array of streamline angles (radians) at each point on the plug surface  
 XAP - the non-dimensional axial location of the plug point just upstream of the plug base.  
 XCP - the non-dimensional axial distance at point #1  
 XINT - the non-dimensional axial distance at the new location of point #1  
 XP - the non-dimensional array of plug axial distances  
 XPF - the non-dimensional value of the axial distance to the plug base  
 X2A - the non-dimensional axial distance of the point just downstream of the characteristic intersection  
 Z13 - the non-dimensional distance between points #1 and #3

#### SOLUTION METHOD

Redefine the shape integer as a real number.

1.  $EPS = NEPS$

Specify the number of iterations.

2.  $NLOOPS = 5$

Determine approximate region into which the characteristic will intersect the plug.

3.  $DO\ 4\ IP = 1, NOPPTS$

4. If  $XP(IP) > X1$ , then GO TO 35

The last plug point will be set. This will occur right at the plug tip. Specify an integer denoting this fact. (See Fig. XIII-2)

5.  $ISURF = 1$

Determine the variables except the Prandtl-Meyer (P-M) angle at the last point.

6.  $T3 = TP(NOPPTS)$

7.  $X3 = XP(NOPPTS)$

8.  $R3 = RP(NOPPTS)$

9.  $S3 = S2$

Redefine other points.

10.  $XAP = X2$

11.  $RAP = R2$

12.  $XCP = X1$

13.  $RCP = R1$

Determine the slope of the non-characteristic line connecting these two points.

14.  $SLAC = (RAP - RCP) / (XAP - XCP)$

Determine the Mach number and corresponding Mach angle at the characteristic point just before the intersection.

15. CALL PMTURN

16.  $PMU = \text{ASIN}(1/PMACH)$

Calculate the slope of the characteristic.

17.  $SLCH = \text{TAN}(T1 - PMU)$

An iteration is begun in which the location of the characteristic point is moved such that the characteristic point on the plug surface falls extremely close to the plug tip.

18. DO 26 IDUM = 1,20

Calculate the intersection point along the previously calculated non-characteristic line.

19.  $XINT = (RAP - 1 + (SLCH)(XPF) - (SLAC)(XAP)) / (SLCH - SLAC)$

Determine the ratio of distances.

20.  $RATIO = (XINT - X2) / (X1 - X2)$

Linearly interpolate for additional variables along the non-characteristic line.

21.  $TINT = T2 + (RATIO)(T1 - T2)$

22.  $NUINT = NU2 + (RATIO)(NU1 - NU2)$

23.  $SINT = S2^* + (RATIO) / (S1 - S2)$

Calculate the Mach number, Mach angle, and slope of the characteristic.

24. CALL PMTURN

25.  $PMU = \text{ASIN}(1/PMACH)$

26.  $SLCH = \text{TAN}(TINT - PMU)$

Calculate the final radius of the intersection.

27.  $RINT = (SLAC)(XINT - XAP) + RAP$

Reset the characteristic point just before the intersection.

28.  $X1 = XINT$

29.  $R1 = RINT$

30.  $NU1 = NUINT$

31.  $T1 = TINT$

32.  $S1 = SINT$

Redefine the Mach angle also.

33.  $MU1 = PMU$

34. GO TO 56

The characteristics may still intersect the plug. Determine the first estimate of the streamline angle. (See Fig. XIII-1)

35.  $T3 = TP(IP + 1)$

36. If  $IP = NOPPTS$ , then  $T3 = TP(IP)$

Determine the Mach number and Mach angle at the characteristic point just before the intersection (point #1).

37. CALL PMTURN

38.  $MU1 = ASIN(1/M1)$

Determine the plug slope.

39.  $SL2 = TAN(T3)$

Calculate the slope of the characteristic.

40.  $SL1 = TAN(T1 - MU1)$

Redefine the plug point.

41.  $R2A = RP(IP + 1)$

42. If  $IP = NOPPTS$ , then  $R2A = RP(IP)$

43.  $X2A = XP(IP + 1)$

44. If  $IP = NOPPTS$ , then  $X2A = XP(IP)$

Begin an iteration procedure to accurately locate the intersection of the characteristic and the plug.

45. DO 55 IDUM = 1, NLOOPS

Calculate the intersection.



$$46. \quad X3 = ((X2A)(SL2) - (X1)(SL1) - (R2A - R1)) / (SL2 - SL1)$$

Make sure the edge of the plug has not been exceeded.

$$47. \quad \text{If } X3 > XPF, \text{ then GO TO 5}$$

Calculate the radius of the plug at the intersection. A second order interpolation of the plug points is used.

$$48. \quad R3 = \text{TAB}(X3, XP, RP, \text{NOPPTS}, 2)$$

Check for error conditions.

$$49. \quad \text{If } X3 < X2, \text{ and } IDUM < NLOOPS, \text{ then } X3 = X2 + 0.001/IDUM$$

$$50. \quad \text{If } X3 < X2, \text{ or } R3 < 0, \text{ then PRINT 104, X3, X2, R3.}$$

$$51. \quad \text{If } R3 < 0, \text{ then GO TO 82}$$

Calculate the streamline angle.

$$52. \quad T3 = \text{TAB}(X3, XP, TP, \text{NOPPTS}, 2)$$

Change variable names, and calculate the slope of the plug at the intersection.

$$53. \quad R2A = R3$$

$$54. \quad SL2 = \text{TAN}(T3)$$

$$55. \quad X2A = X3$$

Calculate the average radius.

$$56. \quad R13 = (R1 + R3)/2$$

Calculate the distance along the characteristic.

$$57. \quad Z13 = \text{SQRT}((R1 - R3)^2 + (X1 - X3)^2)$$

$$58. \quad C1 = ((\text{SIN}(MU1))(\text{SIN}(T1)) / R13) (EPS)$$

Calculate rotational part of the characteristics at point #1.  
(Equation I-35)

$$59. \quad K1 = (\text{SIN}(MU1))(\text{COS}(MU1)) / G$$

Calculate the entropy.

60.  $S3 = S2$

Obtain a first estimate of the Prandtl-Meyer (P-M) angle.

61.  $NU3G = NU1 + T1 - T3 + (C1)(Z13) - (K1)(S3-S1)/RG$

Determine the corresponding Mach number and Mach angle.

62. CALL PMTURN

63.  $MU3G = ASIN(1/M3G)$

Calculate the integrand and rotational effects at the new point.  
(Equation I-34 and I-35)

64.  $C3 = ((SIN(MU3G))(SIN(T3)))/R3(EPS)$

65.  $K3 = (SIN(MU3G))(COS(MU3G))/G$

Begin iteration procedure to find the P-M angle at the new point.

66. DO 74 IDUM = 1,NLOOPS

Calculate the average values of the integrand and rotational components.

67.  $C13 = ((C1)(R13))/R1 + C3)/2$

68.  $K13 = (K1 + K3)/2$

Calculate the isentropic value of the P-M angle.

69.  $F1I = NU1 + (T1 - T3) + (C13)(Z13)$

Include rotational effects. (Equations I-39a and I-40)

70.  $NU3 = F1I - (K13)(S3-S1)/RG$

Determine the new Mach number and Mach angle.

71. CALL PMTURN

72.  $MU3 = ASIN(1/M3)$

Calculate the new values of the integrand and the rotational effects.  
(Equations I-34 and I-35)

73.  $C3 = ((SIN(MU3))(SIN(T3)))/R3(EPS)$

74.  $K3 = (SIN(MU3))(COS(MU3))/G$

75. RETURN

Print out error conditions and return.

76. PRINT 101, NU1

77. RETURN

78. PRINT 102, NU3G

79. RETURN

80. PRINT 103, NU3

81. RETURN

82. RETURN 1

#### XIV. SUBROUTINE CALC

#### XIV. SUBROUTINE CALC

Subroutine CALC calculates an interior method of characteristics point (#3) when two upstream points are given. A right-running characteristic from point #1 and the intersection with a left-running characteristic from point #2 locates point #3. The additional variables are then calculated.

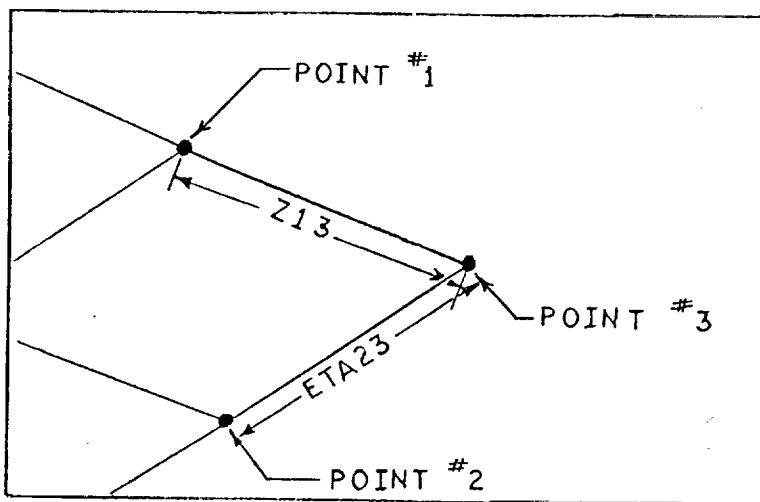


FIGURE XIV-1

#### COMMON BLOCKS

COMMON blocks PLCBLK and POLIP are used.

#### TPNZZL SUBROUTINES

CALC is called by the following subroutines and function subprograms: FBASE6, FLOW, LIPSHK, SSHAPE, and TPNZZL.

CALC calls the subroutine PMTURN.

#### FORTTRAN SYSTEM ROUTINES

Built-in FORTRAN functions ASIN, COS, SIN, SQRT, and TAN are used.

## CALLING SEQUENCE

The calling sequence is:

CALL CALC (\$,NU3,T3,S3,X3,R3,NU1,NU2,T1, T2,S1,S2,X1,X2,R1,R2,G,  
RG,NEPS)

NU3 is the Prandtl-Meyer turn angle at point #3. (See Fig.XIV-1)

T3 is the streamline angle at point #3.

S3 is the entropy at point #3.

X3,R3 are the axial and radial coordinates of point #3.

NU1,NU2 are the Prandtl-Meyer angles for points #1 and #2.

T1,T2 are the streamline angles for points #1 and #2.

S1,S2 are the entropies for points #1 and #2.

X1,X2 are the axial coordinates for points #1 and #2.

R1,R2 are the radial coordinates for points #1 and #2.

G is the ratio of specific heats.

RG is the gas constant.

NEPS =  $\begin{cases} 1 & \text{for axisymmetric flow.} \\ 0 & \text{for planar flow.} \end{cases}$

C1 - the value of the integrand (Equation I-34) at point #1  
 C13 - the average value of the integrand (Equation I-34) between points #1 and #3  
 C2 - the value of the integrand (Equation I-34) at point #2  
 C23 - the average value of the integrand (Equation I-34) between points #2 and #3  
 C3 - the value of the integrand (Equation I-34) at point #3  
 DINT2 - the distance between points #2 and #3  
 DR - a ratio of distances  
 DS - the change in entropy between points #1 and #2  
 DSR - the change in entropy from points #3 to #1  
 D12 - the distance between points #1 and #2  
 EPS - real number (=1.0 for axisymmetric flow; =0.0 for planar flow)  
 ETA23 - the non-dimensional distance between points #2 and #3  
 F1 - Equation I-37a  
 F1G - the first estimate of Equation I-37a  
 F1GI - the isentropic portion of Equation I-37a (first estimate)  
 F1I - the isentropic portion of Equation I-37a  
 F2 - Equation I-37b  
 F2G - the first estimate of Equation I-37b  
 F2GI - the first estimate of the isentropic portion of Equation I-37b  
 F2I - the isentropic portion of Equation I-37b  
 ICROSS - an integer denoting when characteristics of the same family have crossed indicating a shock formation  
 IDUM - an integer counter used in iterating  
 ILOC - an integer which tells what portion of the flow field is being calculated (See Fig. IV-1)  
 K1 - the value of the rotational effects (Equation I-35) at point #1

K13 - the average value of the rotational effects (Equation I-35) between points #1 and #3  
 K2 - the value of the rotational effects (Equation I-35) at point #2  
 K23 - the average value of the rotational effects (Equation I-35) between points #2 and #3  
 K3 - the value of the rotational effects (Equation I-35) at point #3  
 LOC - an integer which tells what portion of the flow field is being calculated (See Fig. I-3)  
 LSHK - an integer which tells whether a "lip shock" is present  
 MU1 - the Mach angle (radians) at point #1  
 MU2 - the Mach angle (radians) at point #2  
 MU3 - the Mach angle (radians) at point #3  
 MU3G - the first estimate of the Mach angle (radians) at point #3  
 M1 - the Mach number at point #1  
 M2 - the Mach number at point #2  
 M3 - the Mach number at point #3  
 M3G - the first estimate of the Mach number at point #3  
 NLOOPS - the number of iterations in the characteristics solution  
 NU3G - the first estimate of the Prandtl-Meyer angle (radians) at point #3  
 POLP - the stagnation pressure ( $\text{lb/in}^2$ ) downstream of the "lip shock"  
 RATIO - inverse of the slope between points #1 and #2  
 RINT - the non-dimensional radial location of point #3  
 R13 - the average non-dimensional radius between points #1 and #3  
 R23 - the average non-dimensional radius between points #2 and #3  
 SL1 - the slope of the right-running characteristic from point #1  
 SL2 - the slope of the left-running characteristic from point #2



- TNTB - the average slope between points #1 and #2
- T3G - the first estimate of the streamline angle (radians) at point #3
- XINT - the non-dimensional axial location of point #3
- Z13 - the non-dimensional distance from points #1 to #3

## SOLUTION METHOD

Change the shape integer to a real number.

1.  $EPS = NEPS$

Set a calculational integer used in determining the crossing of characteristic waves of the same family.

2.  $ICROSS = 0$

Set the number of iterations.

3.  $NLOOPS = 5$

Determine the Mach numbers at the two given points and their respective Mach angles and characteristic slopes.

4. CALL PMTURN

5. CALL PMTURN

6.  $MU1 = ASIN(1/M1)$

7.  $MU2 = ASIN(1/M2)$

8.  $SL1 = TAN(T1 - MU1)$

9.  $SL2 = TAN(T2 + MU2)$

Calculate the location of the intersection of these two characteristic waves.

10.  $X3 = ((X2)(SL2) - (X1)(SL1) - (R2-R1))/(SL2-SL1)$

11.  $R3 = (R1 + (SL1)(X3 - X1))$

Check for crossing of characteristics.

12. If  $X3 < X1$ , and  $R3 > R1$ , then  $ICROSS = 1$

Print out message if characteristics cross.

13. If  $ICROSS = 1$ , then PRINT 104, X3, X1, R3

Check for error conditions.

14. If  $R3 < 0$ , GO TO 70

Look for an intersection with a shock wave.

15. If ICROSS = 1, and ILOC = 4, GO TO 70

16. If ICROSS = 1, and LSHK = 1, GO TO 70

Crossing of characteristics also produces error conditions.

17. If ICROSS = 1, then GO TO 70

Calculate the distance from point #2 to point #3,

18.  $ETA23 = \sqrt{(R3-R2)^2 + (X3-X2)^2}$

Calculate the distance from point #1 to point #3.

19.  $Z13 = \sqrt{(R3-R1)^2 + (X3-X1)^2}$

Determine the average radii.

20.  $R23 = (R2 + R3)/2$

21.  $R13 = (R1 + R3)/2$

Calculate the value of the integrand at points #1 and #2. (Equation I-34)

22.  $C1 = ((\sin(MU1)) (\sin(T1)) / R13) (EPS)$

23.  $C2 = ((\sin(MU2)) (\sin(T2)) / R23) (EPS)$

Calculate the rotational effects at these points. (Equation I-35)

24.  $K1 = (\sin(MU1)) (\cos(MU1)) / G$

25.  $K2 = (\sin(MU2)) (\cos(MU2)) / G$

Collect terms at each point.

26.  $F2GI = NU2 - T2 + (C2) (ETA23)$

27.  $F1GI = NU1 + T1 + (C1) (Z13)$

Calculate the average slope of the streamline angles.

28.  $TNTB = (\tan(T1) + \tan(T2)) / 2$

Calculate the inverse of the slope between points #1 and #2.

29.  $RATIO = (X2-X1) / (R2-R1)$

Make sure the streamline slopes do not add up to be zero.

30. If  $TNTB = 0$ , then GO TO 34

Project back from point #3 to the intersection with the line constructed between points #1 and #2.

31.  $RINT = ((X3 - X1) + (RATIO)(R1) - (R3/TNTB))/(RATIO - (1/TNTB))$

32.  $XINT = ((RINT - R3)/TNTB) + X3$

33. GO TO 36.

The average slope is zero. Locate the intersection.

34.  $RINT = R3$

35.  $XINT = (RATIO)(R3 - R1) + X1$

Calculate the distance between points #1 and #2.

36.  $D12 = \text{SQRT}((R2 - R1)^2 + (X2 - X1)^2)$

Calculate the distance from point #2 to the intersection on the line between points #1 and #2. (See Fig. XIV-1)

37.  $DINT2 = \text{SQRT}((R2 - RINT)^2 + (X2 - XINT)^2)$

Calculate the ratio of these two distances.

38.  $DR = DINT2/D12$

Determine the change in entropy between points #1 and #2.

39.  $DS = S2 - S1$

Calculate the incremental change in entropy at the intersection.

40.  $DSR = (DR)(DS)$

Determine the entropy.

41.  $S3 = S2 - DSR$

Calculate auxiliary equations. (Equations I-39a and I-39b)

42.  $F2G = F2GI - (K2)(S3 - S2)/RG$

43.  $F1G = F1GI - (K1)(S3 - S1)/RG$

Make an initial estimate of the Prandtl-Meyer (P-M) angle and streamline angle at point #3.

$$44. \quad NU3G = (F1G + F2G)/2$$

$$45. \quad T3G = (F1G - F2G)/2$$

Determine the Mach number and corresponding Mach angle at point #3.

$$46. \quad \text{CALL PMTURN}$$

$$47. \quad MU3G = \text{ASIN}(1/M3G)$$

Calculate the integrand and rotational terms at point #3. (Equations I-34 and I-35)

$$48. \quad C3 = ((\text{SIN}(MU3G)) (\text{SIN}(T3G)) / R3) (\text{EPS})$$

$$49. \quad K3 = (\text{SIN}(MU3G)) (\text{COS}(MU3G)) / G$$

Begin iteration procedure to calculate the P-M and streamline angles at point #3.

$$50. \quad \text{DO } 64 \text{ IDUM} = 1, \text{NLOOPS}$$

Calculate the average values of the integrands.

$$51. \quad C13 = ((C1) (R13) / R1 + C3) / 2$$

$$52. \quad C23 = ((C2) (R23) / R2 + C3) / 2$$

Determine the average value of the rotational effects.

$$53. \quad K13 = (K1^* + K3) / 2$$

$$54. \quad K23 = (K2 + K3) / 2$$

Recalculate auxiliary equations.

$$55. \quad F21 = NU2 - T2 + (C23) (\text{ETA}23)$$

$$56. \quad F11 = NU1 + T1 + (C13) (Z13)$$

$$57. \quad F2 = F21 - (K23) (S3 - S2) / RG$$

$$58. \quad F1 = F11 - (K13) (S3 - S1) / RG$$

Calculate the P-M and streamline angles at point #3. (Equations I-36a and I-36b)

$$59. \quad NU3 = (F1 + F2) / 2$$

$$60. \quad T3 = (F1 - F2) / 2$$

Calculate the Mach number and corresponding Mach angle at point #3.

61. CALL PMTURN

62.  $MU3 = \text{ASIN}(1/M3)$

Calculate the integrand and rotational effects terms. (Equations I-34 and I-35)

63.  $C3 = ((\text{SIN}(MU3)) (\text{SIN}(T3))/R3) (EPS)$

64.  $K3 = (\text{SIN}(MU3)) (\text{COS}(MU3))/G$

Return to calling program.

65. Return

Check for error conditions in the calculation of Mach numbers using PMTURN.

66. PRINT 101, NU1

67. Return

68. PRINT 102, NU2

69. Return

70. PRINT 103, NU3

71. Return " .

Characteristics have crossed. Print out message to this effect and make a non-standard return.

72. PRINT 104, X3, X1, R3

73. RETURN 1

XV. SUBROUTINE SURF

## XV. SUBROUTINE SURF

Subroutine SURF is used to calculate the method of characteristics point on a lower solid surface boundary. This is used to calculate the "corresponding inviscid jet boundary" when a conetail solution is employed. In addition, it is used to calculate the far wake boundary point downstream of recompression. This subroutine is also used to turn the corner from the plug to the near wake region even for a constant pressure boundary solution of the near wake.

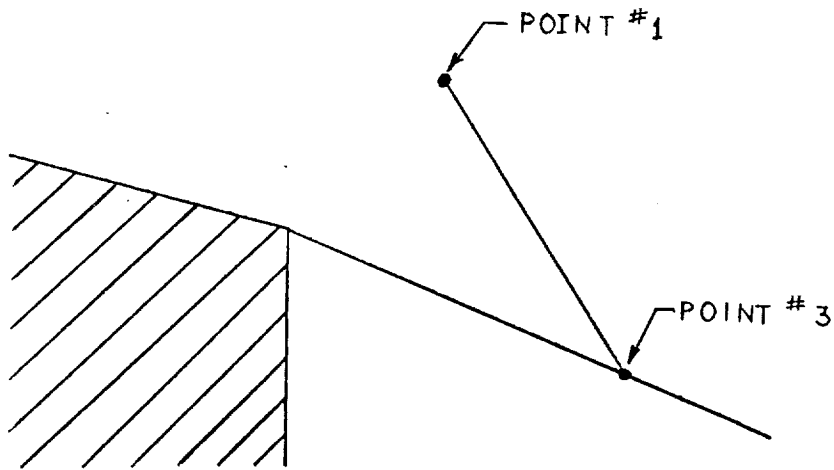


FIGURE XV - 1

### COMMON BLOCKS

COMMON blocks CNRANG, CORNER, PARAM, PLCBLK, and THETBK are used.

### TPNZZL SUBROUTINES

SURF is called by the following subroutines: FLOW, LIPSHK, and SSHAPE.

SURF calls the subroutine PMTURN.

### FORTRAN SYSTEM ROUTINES

Built-in FORTRAN functions ASIN, COS, SIN, SQRT, and TAN are used.



## CALLING SEQUENCE

The calling sequence is:

CALL SURF (\$,NU3,T3,S3,X3,R3,NU1,T1,S1,X1,R1,TS,SB,G,RG,NEPS)

NU3 is the Prandtl-Meyer turn angle on the boundary point.

T3 is the streamline angle of the boundary point.

S3 is the entropy at the boundary point.

X3,R3 are the axial and radial coordinates of the boundary point.

NU1 is the Prandtl-Meyer angle of point #1. (See Fig. XV-1)

T1 is the streamline angle at point #1

S1 is the entropy at point #1

X1,R1 are the axial and radial coordinates of point #1

TS is the streamline angle of the boundary surface.

SB is the boundary entropy

G is the ratio of specific heats

RG is the gas constant.

$$NEPS = \begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$$

# SURF

- B      - y-intercept of characteristic line
- Cl     - the value of the integrand (Equation I-34) at point #1
- Cl3    - the average value of the integrands (Equation I-34) between  
         points #1 and #3
- C3     - the value of the integrand (Equation I-34) at point #3
- C3AS   - the square of the Crocco number adjacent to recompression
- DTC    - the increment of streamline angle (radians) in expanding from  
         the plug to the near wake (See Fig. IV-3)
- D23    - distance at recompression between points #3 and #1
- D23P   - distance at recompression between points #3 and #1
- EPS    - real number (=1.0 for axisymmetric flow; =0.0 for planar flow)
- FlI    - the isentropic portion of Equation I-37a
- I       - subscript
- IDUM   - an integer counter used in iterating
- II      - integer used in counting the number of discrete turns from the  
         plug base to the near wake
- ILOC   - an integer which tells what portion of the flow field is being  
         calculated (see Fig. IV-1)
- K1      - the value of the rotational effects (Equation I-35) at point #1
- K13    - the average value of the rotational effects (Equation I-35)  
         between points #1 and #3
- K3      - the value of rotational effects (Equation I-35) at point #3)
- LOC    - an integer which tells what portion of the flow field is being  
         calculated (See Fig. I-3)
- MU1    - the Mach angle (radians) at point #1
- MU3    - the Mach angle (radians) at point #3
- MU3G   - the first estimate of the Mach angle (radians) at point #3

M1 - the Mach number at point #1  
 M3 - the Mach number at point #3  
 M3G - the initial estimate of the Mach number at point #3  
 NLOOPS - the number of iterations in the characteristics solution  
 NTC - the number of discrete turns in going from the plug surface to the near wake (See Fig. IV-3)  
 NU - array of Prandtl-Meyer turn angles (radians) at each characteristic point  
 NU3G - the first estimate of the Prandtl-Meyer angle (radians) at point #3  
 R - the non-dimensional array of the radius at each characteristic point  
 RTO - ratio of distances at recompression  
 R13 - the average non-dimensional radius between points #1 and #3  
 R2 - the non-dimensional base radius, OR the non-dimensional wake radius ratio  
 R3P - the non-dimensional radius of point #3  
 S - the array of entropy at each characteristic point  
 SL1 - the slope of the right-running characteristic from point #1  
 SL2 - the slope of the boundary  
 T - the array of streamline angles (radians) at each characteristic point  
 THET12 - change in streamline angle (radians) in going from the plug surface to the near wake  
 THET3A - the change in streamline angle (radians) across recompression  
 T2 - the streamline angle (radians) on the surface  
 X - array of axial coordinates (non-dimensional) at each characteristic point  
 X2 - the non-dimensional axial location of the base, or the non-dimensional axial location of recompression

- X3P - the non-dimensional axial location of point #3
- Z13 - the non-dimensional distance between points #1 and #3

## SOLUTION METHOD

Change the shape integer to a real number.

1.  $EPS = NEPS$

Determine the streamline angle.

2.  $T3 = TS$

Set the number of iterations.

3.  $NLOOPS = 5$

Calculate the Mach number and Mach angle at the point in the flow field (Point #1) whose right-running characteristic intersects the boundary.

4.  $CALL PMTURN$

5.  $MU1 = ASIN(1/M1)$

Define a calculational integer.

6.  $II = I/2$

Redefine the streamline angle on the surface (point #3) if an expansion about the plug tip is taking place.

7. If  $TS < 0$ , and  $II < NTC$ , and  $ILOC = 1$ , then  $T3 = (II)(DTC) + T2$

See if a compressive turn is required.

8. If  $THET12 < 0$ , then  $T3 = TS$

Calculate the slope of the angle of the boundary.

9.  $SL2 = TAN(TS)$

Again take into account the possibility of the solution moving through the expansion.

10. If  $TS < 0$ , and  $II < NTC$ , and  $ILOC = 1$ , then  $SL2 = TAN(T3-DTC/2)$

Look for a compression.

11. If  $THET12 < 0$ , then  $SL2 = TAN(TS)$

Calculate the slope of the characteristic at point #1.

$$12. \quad SL1 = \tan(T1 - MU1)$$

Determine the intersection of the characteristic and the boundary.

$$13. \quad X3 = ((X2)(SL2) - (X1)(SL1) - (R2 - R1)) / (SL2 - SL1)$$

$$14. \quad R3 = R1 + (SL1)(X3 - X1)$$

Redefine this intersection point.

$$15. \quad X3P = X3$$

$$16. \quad R3P = R3$$

Check for crossing of characteristics. Print Message.

$$17. \quad \text{If } X3 < X1 \text{ or } R3 < 0, \text{ PRINT } 104, X3, X1, R3$$

$$18. \quad \text{If } R3 < 0, \text{ GO TO } 47$$

$$19. \quad \text{If } X3 < X1, \text{ GO TO } 47$$

Calculate the average radius between points #1 and #3. (See Fig. XV-1)

$$20. \quad R13 = (R1 + R3) / 2$$

Determine the distance between these two points.

$$21. \quad Z13 = \sqrt{(R1 - R3)^2 + (X1 - X3)^2}$$

Calculate the integrand at point #1. (Equation I-34)

$$22. \quad C1 = ((\sin(MU1))(\sin(T1)) / R13)(EPS)$$

Determine rotational effects at point #1. (Equation I-35)

$$23. \quad K1 = (\sin(MU1))(\cos(MU1)) / G$$

The entropy along the boundary remains constant.

$$24. \quad S3 = SB$$

Obtain a first estimate of the Prandtl-Meyer (P-M) angle on the surface. (Equation I-40)

$$25. \quad NU3G = NU1 + T1 - T3 + (C1)(Z13) - (K1)(S3 - S1) / RG$$

Determine the corresponding Mach number and Mach angle at point #3.

$$26. \quad \text{CALL PMTURN}$$

27.  $MU3G = \text{ASIN}(1/M3G)$

Calculate the integrand and rotational effects at point #3. (Equations I-34 and I-35)

28.  $C3 = ((\text{SIN}(MU3G)) (\text{SIN}(T3)) / R3) (EPS)$

29.  $K3 = (\text{SIN}(MU3G)) (\text{COS}(MU3G)) / G$

Begin iteration scheme to determine the P-M angle.

30. DO 39 IDUM = 1, NLOOPS

Calculate the average value of the integrand and rotational terms.

31.  $C13 = ((C1) (R13) / R1 + C3) / 2$

32.  $K13 = (K1 + K3) / 2$

Calculate the isentropic value of the P-M angle.

33.  $F1I = NU1 + (T1 - T3) + (C13) (Z13)$

Add in rotational effects. (Equations I-39a and I-40)

34.  $NU3 = F1I - (K13) (S3 - S1) / RG$

Check for possible error and rectify.

35. If  $NU3 < 0$ , then  $NU3 = NU3G$

Determine the corresponding Mach number and Mach angle at point #3.

36. CALL PMTURN

37.  $MU3 = \text{ASIN}(1/M3)$

Calculate integrand and rotational terms at point #3. (Equations I-34 and I-35)

38.  $C3 = ((\text{SIN}(MU3)) (\text{SIN}(T3)) / R3) (EPS)$

39.  $K3 = (\text{SIN}(MU3)) (\text{COS}(MU3)) / G$

Return to calling program.

40. RETURN

Print out error conditions encountered in calculating Mach numbers.  
Return to calling program.

41. PRINT 101, NU1

42. RETURN

43. PRINT 102, NU3G

44. RETURN

45. PRINT 103, NU3

46. RETURN

Check for the possibility that a too-coarse grid resulted in  $R3 < 0$ .

47. If  $R3 < 0$ , and  $X3 > X1$ , then GO TO 49

This is not the case, so make a non-standard return.

48. RETURN 1

Point #1 will now be reset in an attempt to rectify the problem of a too-coarse grid. First determine the Y-intercept of the point along the boundary.

49.  $B = R3 - (SL2)(X3)$

Reset the new value of R3.

50.  $R3 = 0.0300$

Calculate the corresponding value of X.

51.  $X3 = (R3 - B) / SL2$

Calculate the distance between the previous boundary point and the old location of point #3.

52.  $D23P = \sqrt{(X3P - X(I-1,1))^2 + (R3P - R(I-1,1))^2}$

Calculate the distance between the previous boundary point and the new location of point #3.

53.  $D23 = \sqrt{(X3 - X(I-1,1))^2 + (R3 - R(I-1,1))^2}$

Take a ratio of these distances.

54.  $RTO = D23 / D23P$

Reset point #1.

55.  $X1 = X(I-1,1) + (RTO)(X1 - X(I-1,1))$



56.  $R1 = R(I-1,1) + (RTO)(R1-R(I-1,1))$   
57.  $NU1 = NU(I-1,1) + (RTO)(NU1-NU(I-1,1))$   
58.  $T1 = T(I-1,1) + (RTO)(T1-T(I-1,1))$   
59.  $S1 = S(I-1,1) + (RTO)(S1-S(I-1,1))$

Reset characteristic matrix points.

60.  $X(I,2) = X1$   
61.  $R(I,2) = R1$   
62.  $NU(I,2) = NU1$   
63.  $T(I,2) = T1$   
64.  $S(I,2) = S1$

Repeat calculations.

65. GO TO 4

XVI. SUBROUTINE CPB

## XVI. SUBROUTINE CPB

Subroutine CPB is used to calculate the method of characteristics point on a constant pressure boundary. This may be either the "corresponding inviscid jet boundary" or the external free jet boundary.

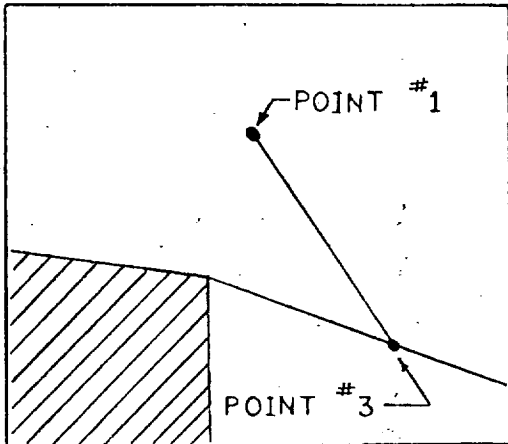


FIGURE XVI-1

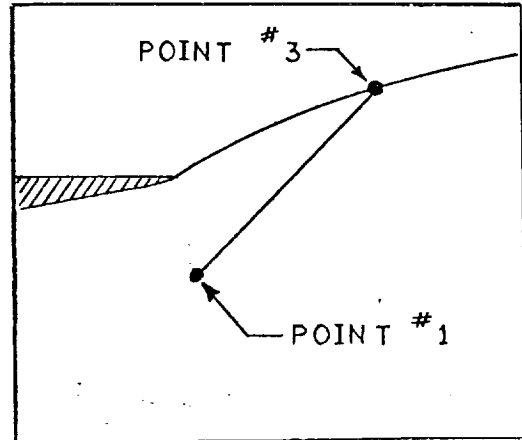


FIGURE XVI-2

### COMMON BLOCKS

No COMMON blocks are used.

### TPNZZL SUBROUTINES

Subroutines FLOW and LIPSHK and function subprogram FBASE6 call CPB.

CPB calls the subroutine PMANGL

### FORTRAN SYSTEM ROUTINES

Built-in FORTRAN functions ASIN, COS, SIN, SQRT, and TAN are used.

## CALLING SEQUENCE

The calling sequence is:

```
CALL CPB($,NU3,T3,S3,X3,R3,NU1,T1,S1,X1,R1,NUP,TP,XP,RP,G,RG,  
N,NEPS)
```

NU3 is the Prandtl-Meyer angle on the boundary (point #3).

T3 is the streamline angle at the boundary.

S3 is the entropy at the boundary.

X3,R3 are the axial and radial coordinates of the boundary point.

NU1 is the Prandtl-Meyer angle of point #1. (See Fig. XVI-1)

T1 is the streamline angle of point #1.

S1 is the entropy at point #1.

X1,R1 are the axial and radial coordinates of point #1.

NUP is the Prandtl-Meyer turn angle of the first upstream boundary point.

TP is the streamline angle of that point.

XP,RP are the axial and radial coordinates of the first upstream boundary point.

G is the ratio of specific heats.

RG is the gas constant

$$N = \begin{cases} 1 & \text{for an upper boundary} \\ 0 & \text{for a lower boundary} \end{cases}$$

$$NEPS = \begin{cases} 1 & \text{for axisymmetric flow} \\ 0 & \text{for planar flow} \end{cases}$$

C13    - the average value of the integrands (Equation I-34) between points #1 and #3  
 C3     - the value of the integrand (Equation I-34) at point #3  
 EPS    - real number (=1.0 for axisymmetric flow; = 0.0 for planar flow)  
 IDUM   - an integer counter used in iterating  
 K13    - the average value of the rotational effects (Equation I-35) between points #1 and #3  
 K3     - the value of the rotational effects (Equation I-35) at point #3  
 MU1    - the Mach angle (radians) at point #1  
 MU3G   - the first estimate of the Mach angle (radians) at point #3  
 M1     - the Mach number at point #1  
 M3G    - the first estimate of the Mach number at point #3  
 NLOOPS - the number of iterations in the characteristics solution  
 R13    - the average non-dimensional radius between points #1 and #3  
 SL1    - the slope of the characteristic from point #1 to the boundary  
 SL2    - the slope of the boundary  
 TP3AV   - an average streamline angle (radians) of the boundary at the intersection with the characteristic  
 Z13    - the non-dimensional distance between points #1 and #3

## SOLUTION METHOD

Change the shape integer to a real number.

1.  $EPS = NEPS$

Set the number of iterations.

2.  $NLOOPS = 5$

On a constant pressure boundary, the Prandtl-Meyer (P-M) angle remains constant, as does the entropy.

3.  $NU3 = NUP$

4.  $S3 = SP$

Point #3 is the point to be calculated on the boundary, whereas point #1 is the point whose characteristic intersects the boundary. Now determine the Mach numbers at points #3 and #1. (See Fig. XVI-1)

5. CALL PMTURN

6. CALL PMTURN

Calculate the respective Mach angles.

7.  $MU3G = \text{ASIN}(1/M3G)$

8.  $MU1 = \text{ASIN}(1/M1)$

Assume for the present that the average streamline angle at the boundary is equal to the value at the previous boundary point.

9.  $TP3AV = TP$

Calculate the slope of the characteristic for a lower boundary.

10.  $SL1 = \text{TAN}(T1 - MU1)$

The slope is different for an upper boundary.

11. If  $N = 1$ , then  $SL1 = \text{TAN}(T1 + MU1)$

All variables except P-M angle and entropy become iterative in a constant pressure boundary solution.

12. DO 30 IDUM = 1, NLOOPS

Calculate the average slope of the boundary.

$$13. \quad SL2 = \text{TAN}(TP3AV)$$

Determine the intersection of the boundary and the characteristic wave.

$$14. \quad X3 = ((XP)(SL2) - (X1)(SL1) - (RP-R1))/(SL2-SL1)$$

$$15. \quad R3 = R1 + (SL1)(X3-X1)$$

Check for possible error; Print out message.

16. If  $X3 < XP$ , or  $R3 < 0$ , then PRINT 103, X3, XP, R3.

17. If  $R3 < 0$ , then GO TO 36

18. If  $X3 < XP$ , then GO TO 36

Calculate an average radius.

$$19. \quad R13 = (R1 + R3)/2$$

Determine the distance between points #1 and #3.

$$20. \quad Z13 = \text{SQRT}((R1-R3)^2 + (X1-X3)^2)$$

On the first iteration, additional calculations must be made.

21. If IDUM > 1, then GO TO 24

Calculate the integrand. (Equation I-34)

$$22. \quad C13 = ((\text{SIN}(\text{MU1}))(\text{SIN}(T1)))/R13(\text{EPS})$$

Calculate the rotational term. (Equation I-35)

$$23. \quad K13 = (\text{SIN}(\text{MU1}))(\text{COS}(\text{MU1}))/G$$

Calculate the streamline angle at point #3. (Equations I-39a and I-41)

$$24. \quad T3 = \text{NU1} + T1 - \text{NU3} + (C13)(Z13) - (K13)(S3-S1)/RG$$

For an upper boundary the streamline angle becomes: (See Fig. XVI-2)

$$25. \quad \text{If } N = 1, \text{ then } T3 = \text{NU3} - \text{NU1} + T1 - (C13)(Z13) + (K13)(S3-S1)/RG.$$

Calculate the integrand at point #3 and the average value between points #1 and #3. (Equations I-34)

$$26. \quad C3 = ((\text{SIN}(\text{MU3G}))(\text{SIN}(T3)))/R3(\text{EPS})$$

$$27. \quad C13 = ((C13) (R13)/R1 + C3)/2$$

Calculate the rotational term at point #3 and the average value between points #1 and #3. (Equation I-35)

$$28. \quad K3 = (\sin(\mu3G)) (\cos(\mu3G))/G$$

$$29. \quad K13 = (K13 + K3)/2$$

Calculate the average streamline angle between point #3 and the previous boundary point.

$$30. \quad TP3AV = (T3 + TP)/2$$

Return to calling program.

31. RETURN

Print out error conditions in Mach number calculations and return to calling program.

32. PRINT 101, NU3

33. RETURN

34. PRINT 102, NU1

35. RETURN

A crossing of characteristic waves is indicated.

36. RETURN 1



XVII. SUBROUTINE STRLNE

## XVII. SUBROUTINE STRLNE

Subroutine STRLNE calculates up to 10 streamlines in a given flow field. This subroutine sets the locations of the initial point of a streamline along the initial data line at equally spaced intervals. The subroutine SEARCH locates the position of the point in the characteristics mesh. (See Fig. XVII-1)

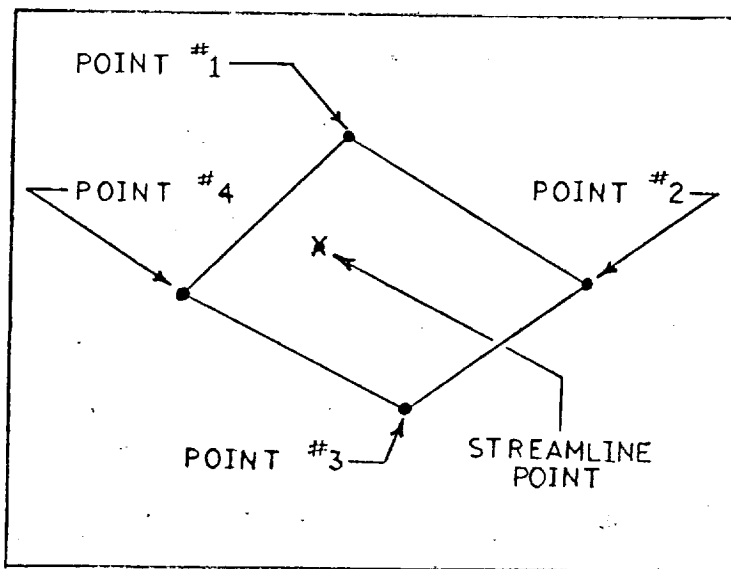


FIGURE XVII-1

### COMMON BLOCKS

COMMON blocks CH1BLK, CNTR, GAS, PARAM, PLCBLK, PTNOS, SIZE, STRBLK, STRL, STRSRT and TPN are used.

### TPNZL SUBROUTINES

Subroutine STRLNE may be called by the following subroutines: FLOW, LIPSHK, SETUP, and SSHAPE. Also the function subprogram FBASE6 calls STRLNE.

STRLNE calls subroutines and functions PMANGL, PMTURN, SEARCH, and TAB.

## FORTRAN SYSTEM ROUTINES

Built-in FORTRAN functions SQRT and TAN are used.

## CALLING SEQUENCE

The calling sequence is:

CALL STRLNE (NDF)

NDF is the value of the last axial subscript in a characteristic matrix.

A1 - real number ( = 0.0 on an upper surface; = 1.0 for all other points)  
 A2 - real number ( = 0.0 on a right surface; = 1.0 for all other points)  
 A3 - real number ( = 0.0 on a lower surface; = 1.0 for all other points)  
 A4 - real number ( = 0.0 on a left surface; = 1.0 for all other points)  
 D - non-dimensional distance along the initial line  
 DELTR - non-dimensional gap distance at the initial line  
 DH - incremental radial locations of the streamlines  
 DI - non-dimensional distance along the initial line (array)  
 DINC - incremental distance along the streamline  
 DINCS - incremental distance along the streamline  
 DIST - linear distance between points on the initial line  
 DIT - sum of the distances between the characteristic points and the streamline point  
 DLINCR - the change in streamline incremental distance between the last and the first streamline  
 DTOT - total distance along the initial line  
 D1 - non-dimensional distance between point #1 and the streamline point  
 D2 - distance between point #2 and the streamline point  
 D3 - distance between point #3 and the streamline point  
 D4 - distance between point #4 and the streamline point  
 G - ratio of specific heats  
 I - subscript  
 IEND - last value of the I-subscript  
 ILOC - integer which tells what portion of the flow field is being calculated (See Fig. IV-1)

INCR - array of non-dimensional incremental distances along each streamline  
 INSB - number of subincrements between each incremental distance along each streamline  
 IOPTR - integer describing the type of initial line input  
 J - subscript  
 K - subscript  
 LOC - integer which tells what portion of the flow field is being calculated (See Fig. I-3)  
 MSTR - array of Mach numbers along a streamline  
 NDI - maximum number of I-subscripts in the characteristics matrix  
 NDJ - maximum number of J-subscripts in the characteristics matrix  
 NOIPTS - number of initial points along the starting line  
 NOIPML - NOIPTS - 1  
 NOPPTS - number of plug points  
 NOSPTS - number of streamline points  
 NOSTRL - number of streamlines  
 NSTRL1 - NOSTRL - 1  
 NSTRT - array of integers telling the starting point on each streamline  
 NSTRT1 - the starting point of a streamline  
 NU - array of Prandtl-Meyer turn angles (radians) at each point in the characteristic matrix  
 NUI - array of Prandtl-Meyer angles (radians) at each point along the starting line  
 NUSTR - Prandtl-Meyer angle (radians) at a streamline point  
 NUSTRL - Prandtl-Meyer angle (radians) at a streamline point  
 NU1 - Prandtl-Meyer angle (radians) at point #1  
 NU2 - Prandtl-Meyer angle (radians) at point #2

NU3 - Prandtl-Meyer angle (radians) at point #3  
 NU4 - Prandtl-Meyer angle (radians) at point #4  
 PO1 - chamber stagnation pressure ( $\text{lb/in}^2$ )  
 R - array of non-dimensional radial coordinates at each point  
     in the characteristic matrix  
 RATIO - ratio of axial distances  
 RG - gas constant ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )  
 RI - array of non-dimensional radial coordinates along the starting  
     line  
 RP - non-dimensional array of radial coordinates at each point  
     on the plug surface  
 RPB - plug base radius (inches)  
 RSTR - array of non-dimensional radial coordinates of each streamline  
     point  
 RSTRL - non-dimensional radial coordinate of a streamline point  
 R1 - non-dimensional radius at point #1  
 R2 - non-dimensional radius at point #2  
 R3 - non-dimensional radius at point #3  
 R4 - non-dimensional radius at point #4  
 S - array of entropies at each point in the characteristics matrix  
     ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )  
 SBIN - real number (equivalent to INSB)  
 SI - array of entropies along the initial line ( $\text{ft-lb}_f/\text{lb}_m\text{-}^\circ\text{R}$ )  
 SLTH - array of the slope of the streamline angles at a streamline  
     point  
 SPTS - real number (equivalent to NOSPTS)  
 STRLN - streamline number  
 STRLT - total number of streamlines

T - array of streamline angles (radians) at each point in the characteristic matrix  
 TI - array of streamline angles (radians) along the starting line  
 TO1 - chamber stagnation temperature ( $^{\circ}\text{R}$ )  
 TP - array of streamline angles (radians) at each point on the plug surface  
 TSL - streamline angle (radians) at a point on a streamline  
 TSTR - array of streamline angles (radians) at each point along the streamlines  
 TSTRL - streamline angle (radians) at a streamline point  
 T1 - streamline angle (radians) at point #1  
 T2 - streamline angle (radians) at point #2  
 T3 - streamline angle (radians) at point #3  
 T4 - streamline angle (radians) at point #4  
 X - array of non-dimensional axial coordinates at each point in the characteristics matrix  
 XI - array of non-dimensional axial coordinates at each point along the starting line  
 XL - a non-dimensional axial coordinate of a point on the first streamline  
 XP - array of non-dimensional axial coordinates at each point along the plug surface  
 XPF - non-dimensional axial location of the plug base  
 XSTR - array of non-dimensional axial coordinates at each point along the streamlines  
 X1 - non-dimensional axial coordinate of point #1  
 X2 - non-dimensional axial coordinate of point #2  
 X3 - non-dimensional axial coordinate of point #3  
 X4 - non-dimensional axial coordinate of point #4

## SOLUTION METHOD

See whether any streamlines are to be calculated.

1. If NOSTRL = 0, then return

The first section deals with setting up the initial streamline locations, and is, therefore, only used for the first time STRLNE is called.

2. If LOC > 0 and IOPTR = 2, then GO TO 39

3. If LOC > 1 and IOPTR = 1, then GO TO 39

Set the maximum number of streamline points and redefine as a real number.

4. NOSPTS = 50

5. SPTS = NOSPTS

Determine a calculational integer.

6. NOIPM1 = NOIPTS - 1

Calculate the linear distance along the initial line for each point.

7. DI(1) = 0.00

8. DO 11 I = 2, NOIPTS

9.  $DIST = \sqrt{(XI(I) - XI(I-1))^2 + (RI(I) - RI(I-1))^2}$

10.  $DI(I) = DI(I-1) + DIST$

Calculate the total distance.

11.  $DTOT = DI(I)$

Define another calculational integer.

12. NSTRL1 = NOSTRL - 1

Calculate an increment resulting in equal intervals.

13.  $DH = DTOT / NSTRL1$

Set the first point on the plug surface.



14. XSTR(1,1) = XI(1)

15. RSTR(1,1) = RI(1)

16. CALL PMTURN

17. TSTR(1,1) = TI(1)

Now set up the initial streamline locations along the initial data line. They occur at equal intervals.

18. D = 0.00

19. DO 25 I = 2,NOSTRL

Increment distance.

20. D = D + DH

Linearly interpolate along starting line for streamline variables.

21. XSTR(1,1) = TAB(D,DI,XI,NOIPTS,1)

22. RSTR(1,1) = TAB(D,DI,RI,NOIPTS,1)

23. TSTR(1,1) = TAB(D,DI,TI,NOIPTS,1)

24. NUSTR = TAB(D,DI,NUI,NOIPTS,1)

Change Prandtl-Meyer (P-M) angle to the corresponding Mach number.

25. CALL PMTURN

Now set up increments along each streamline. First, calculate the linear distance across the initial data line.

26. DELTR = SQRT((XI(NOIPTS)-XI(1))<sup>2</sup> + (RI(NOIPTS) - RI(1))<sup>2</sup>)

Set the increment along the first streamline.

27. INCR(1) = (XP(NOPPTS) - XP(1) + 4.0)/SPTS

Set the increment on the last streamline.

28. INCR(NOSTRL) = (4.0)(INCR(1))

29. DLINCR = INCR(NOSTRL) - INCR(1)

Calculate the increment on remaining streamlines.

30. DO 33 I = 2,NSTRL1

Change the streamline to a real number.

31. STRLN = I

Change the total number of streamlines to a real number.

32. STRLT = NOSTRL

Calculate the increment on the streamline.

33. INCR(I) = INCR(1) + (STRLN/STRLT) (DLINCR)

Set the axial locations of all streamline points on the first and last streamlines.

34. DO 36 I = 2,NOSPTS

35. XSTR(1,I) = XSTR(1,I-1) + INCR(1)

36. XSTR(NOSTRL,I) = XSTR(NOSTRL, I-1) + INCR(NOSTRL)

set the starting point on each streamline.

37. DO 38 I = 1,NOSTRL

38. NSTRT(I) = 2

Locate the streamline points on the upper boundary.

39. CONTINUE

For LOC = 0, the upper and lower boundaries are not included in the characteristic matrix.

40. If LOC = 0, then GO TO 85

Define the starting point as a non-subscripted variable.

41. NSTRT1 = NSTRT(NOSTRL)

Begin calculations along the upper boundary.

42. DO 54 I = NSTRT1,NOSPTS

43. DO 46 K = 3,NDF,2

See if the characteristic boundary point is further downstream than the streamline point.

44. If  $X(K,NDJ) < XSTR(NOSTRL,I)$  then GO TO 46

The characteristic boundary point is beyond the streamline point.

45. GO TO 48

46. CONTINUE

No solution is found in the existing matrix.

47. GO TO 55

Linearly interpolate the variables at the streamline point.

First, calculate a ratio of distances to aid in the interpolation.

48.  $RATIO = (X(K,NDJ) - XSTR(NOSTRL,I)) / (X(K,NDJ) - X(K-2,NDJ))$

Calculate variables.

49.  $RSTR(NOSTRL,I) = R(K,NDJ) - (RATIO) (R(K,NDJ) - R(K-2,NDJ))$

50.  $TSTR(NOSTRL,I) = T(K,NDJ) - (RATIO) (T(K,NDJ) - T(K-2,NDJ))$

51.  $NUSTRL = NU(K,NDJ) - (RATIO) (NU(K,NDJ) - NU(K-2,NDJ))$

52. CALL PMTURN

Reset the starting point of this streamline.

53.  $NSTRT(NOSTRL) = I + 1$

54. CONTINUE

Now calculate the streamline points on the first streamline.

When  $LOC > 3$ , no lower boundary streamline will be calculated.

55. If  $LOC > 3$ , then GO TO 85

Define the starting point as a non-subscripted variable.

56.  $NSTRT1 = NSTRT(1)$

Begin searching for the characteristic boundary points which include the streamline point.

57. DO 84 I = NSTRT1,NOSPTS

Make sure the number of streamline points has not exceeded the maximum value.

58. If  $I \geq \text{NOSPTS}$ , then GO TO 63

Locate the characteristic point which is just downstream of the streamline point.

59. DO 62 K = 3, NDF, 2

60. If  $(X(K,1) < XSTR(1,I))$ , then GO TO 62

The point has been found.

61. GO TO 73

62. CONTINUE

The array size of the streamlines has been exceeded. Therefore, let the last point be the last streamline point which has been found. Set the subscript.

63. K = NDF

Check for an incomplete characteristic matrix.

64. If  $X(K,1) < 10^{-6}$ , then GO TO 85

Set the last streamline point.

65.  $XSTR(1,1) = X(K,1)$

66.  $RSTR(1,1) = R(K,1)$

67.  $TSTR(1,1) = T(K,1)$

68.  $NUSTR1 = NU(K,1)$

69. CALL PMTURN

Reset the starting point on this streamline.

70.  $NSTR1(1) = I + 1$

71. If  $NSTR1(1) > \text{NOSPTS}$ , then  $NSTR1(1) = \text{NOSPTS}$

72. GO TO 85

A point has been found. Linearly interpolate for the variables.

73.  $\text{RATIO} = (X(K,1) - XSTR(1,I)) / (X(K,1) - X(K-2,1))$

See if the streamline is still on the plug surface.

74. If  $XSTR(1,I) < XPF$ , then GO TO 78

The streamline is in the near wake region.

75.  $RSTR(1,I) = R(K,1) - (RATIO)(R(K,1) - R(K-2,1))$

76.  $TSTR(1,I) = T(K,1) - (RATIO)(T(K,1) - T(K-2,1))$

77. GO TO 81

The streamline is still on the plug surface. Redefine the axial location of the streamline point.

78.  $XL = XSTR(1,I)$

Use a second order interpolation to find the radius and streamline angle at this point.

79.  $RSTR(1,I) = TAB(XL,XP,RP,NOPPTS,2)$

80.  $TSTR(1,I) = TAB(XL,XP,TP,NOPPTS,2)$

Calculate the P-M angle by a linear interpolation.

81.  $NUSTR1 = NU(K,1) - (RATIO)(NU(K,1) - NU(K-2,1))$

Obtain the corresponding Mach number.

82. CALL PMTURN

Reset the starting point.

83.  $NSTR1(1) = I + 1$

84. CONTINUE

85. CONTINUE

Now determine the variables along intermediate streamlines.

86. DO 125 I = 2, NSTR1

Redefine the starting point as a non-subscripted variable.

87.  $NSTR1 = NSTR(I)$

Now search along each streamline.

88. DO 124 J = NSTR1, NOSPTS

See whether the streamline array is filled.

89. If NSTRT(I) > NOSPTS, then GO TO 124

Redefine the previous streamline point.

90. RSTRL = RSTR(I,J-1)

91. TSTRL = TSTR(I,J-1)

92. XSTRL = XSTR(I,J-1)

93. NUSTRL = PMANGL (MSTR(I,J-1),G)

A total of INSB subincrements are calculated in order to more accurately define the location of each streamline.

94. INSB = 5

Change this variable to a real number.

95. SBIN = INSB

Calculate the subincrement size.

96. DINC = INCR(I)/SBIN

Set the first radius point.

97. RSTRL(1) = RSTR(I,J-1)

Set the slope of the streamline angle for the first subincrement.

98. SLTH(1) = 0.00

Find the position in the characteristics mesh in which the streamline point is located.

99. DO 114 K = 1,INSB

Redefine the subincrement size.

100. DINCS = DINC

The integer IEND is zero when a solution is found and equals unity when the streamline point is not found in the characteristic matrix.

101. IEND = 0

Increment the value of X along the streamline.

102. XSTRL = XSTRL + DINCS

Use a second order expansion to locate the value of R on the streamline.

$$103. \quad RSTRL(K+1) = RSTRL(K) + (DINCS) (TAN(TSTRL)) + (0.5) (DINCS^2) (SLTH(K))$$

Locate the point in the characteristic matrix.

104. CALL SEARCH

Calculate the distances from the streamline coordinate to the individual characteristic points.

$$105. \quad D1 = \text{SQRT}((XSTRL - X1)^2 + (RSTRL(K+1) - R1)^2)$$

$$106. \quad D2 = \text{SQRT}((XSTRL - X2)^2 + (RSTRL(K+1) - R2)^2)$$

$$107. \quad D3 = \text{SQRT}((XSTRL - X3)^2 + (RSTRL(K+1) - R3)^2)$$

$$108. \quad D4 = \text{SQRT}((XSTRL - X4)^2 + (RSTRL(K+1) - R4)^2)$$

An inverse variation with distance is assumed on the variables. Calculate the denominator.

$$109. \quad DIT = (A1/D1) + (A2/D2) + (A3/D3) + (A4/D4)$$

Calculate the P-M angle and streamline angle at the streamline point.

$$110. \quad NUSTRL = (((A1)(NU1)/D1) + ((A2)(NU2)/D2) + ((A3)(NU3)/D3) + ((A4)(NU4)/D4))/DIT$$

$$111. \quad TSL = (((A1)(T1)/D1) + ((A2)(T2)/D2) + ((A3)(T3)/D3) + ((A4)(T4)/D4))/DIT$$

Calculate the slope of the streamline angle.

$$112. \quad SLTH(K+1) = TAN((TSL - TSTRL)/DINCS)$$

Redefine the streamline angle.

$$113. \quad TSTRL = TSL$$

114. CONTINUE

Check to see if any subincrements were calculated.

115. If IEND = 1, and K = 1, then GO TO 125

Save the last value of streamline variables. Make sure all subincrements were calculated.

116.  $XSTR(I,J) = XSTRL$   
117. If  $IEND = 1$ , then  $XSTR(I,J) = XSTRL - DINCS$   
118.  $RSTR(I,J) = RSTRL(K+1)$   
119. If  $IEND = 1$ , then  $RSTR(I,J) = RSTRL(K)$   
120.  $TSTR(I,J) = TSTRL$   
Obtain the corresponding Mach number.  
121. CALL PMTURN  
Reset the starting point.  
122.  $ISTRT(I) = J + 1$   
123. If  $IEND = 1$ , GO TO 125  
124. CONTINUE  
125. RETURN



XVIII. SUBROUTINE PMTURN

### XVIII. SUBROUTINE PMTURN

Subroutine PMTURN solves the Prandtl-Meyer (P-M) turn equation of an ideal gas compressible flow. It either returns the downstream Mach number after an isentropic turn or simply determines the Mach number for a given P-M angle.

A technique similar to the Newton-Raphson method is employed.

#### COMMON BLOCKS

No COMMON blocks are used.

#### TPNZZL SUBROUTINES

The following subroutines call PMTURN: BLAYER, CALC, CPB, FLOW, LIPSHK, SSHAPE, STRLNE, SURF, SURFP, SURFSK.

PMTURN does not call any subroutines or function subprograms.

#### FORTRAN SYSTEM ROUTINES

Built-in FORTRAN functions ATAN and SQRT are used.

#### CALLING SEQUENCE

The calling sequence is:

CALL PMTURN (X,G,THETA,AC,M2,\$)

X is the upstream Mach number OR the value of the Prandtl-Meyer angle

G is the ratio of specific heats

THETA is the isentropic turning angle (degrees)

AC is the accuracy desired

M2 is the Mach number corresponding to a particular Prandtl-Meyer angle.

Usage #1 If the accuracy is negative (e.g.,  $AC = -0.001$ ), then  $X$  is taken as the value of the Prandtl-Meyer angle. The value returned to  $M2$  is then simply the Mach number corresponding to that Prandtl-Meyer angle. For this usage,  $THETA$  is ignored.

Usage #2 If the accuracy is positive (e.g.,  $AC = + 0.001$ ), then  $X$  corresponds to the upstream Mach number. The final (downstream) Mach number,  $M2$ , is returned as the solution. (Note that  $THETA < 0$  for a compression, and  $THETA > 0$  for an expansion.)

ACC - accuracy requirement desired  
 B - argument of the Prandtl-Meyer turn angle equation  
 BETA - difference between the actual and the calculated Prandtl-Meyer angles  
 BETA1 - a Mach number function  
 BETA2 - a ratio of specific heats function  
 DIFF - difference between BETA and BETA2  
 DNU - a portion of the Prandtl-Meyer turn angle equation  
 G1 - an expression containing the ratio of specific heats  
 G2 - the inverse of G1  
 L - integer which counts the number of iterations  
 M1 - Mach number upstream of the expansion  
 NU - equation for the Prandtl-Meyer turn angle  
 NUMAX - maximum Prandtl-Meyer angle (radians)  
 NU1 - Prandtl-Meyer angle (radians) evaluated at the curve's inflection point  
 NU2 - Prandtl-Meyer angle after the expansion (radians)  
 THETAR - change in streamline angle (radians) through the expansion (or isentropic compression)

## SOLUTION METHOD

Define two functions.

$$1. \quad \text{NU}(B) = (G2) (\text{ATAN}((G1)(B))) - \text{ATAN}(B)$$

$$2. \quad \text{DNU}(B) = 1/(1 + ((G1)(B))^2) - 1/(1 + B^2)$$

Define the initial Mach number on P-M angle.

$$3. \quad M1 = X$$

Define two ratios of G.

$$4. \quad G1 = \text{SQRT} ((G-1)/(G + 1))$$

$$5. \quad G2 = 1/G1$$

Calculate the maximum P-M angle.

$$6. \quad \text{NUMAX} = (G2 - 1) (1.57079633)$$

The accuracy requirement determines the type of solution desired.  
If AC is negative, the value of X is taken as the P-M angle.  
The value returned is simply the Mach number corresponding to that P-M angle.

$$7. \quad \text{If } AC < 0, \text{ GO TO } 10$$

$$8. \quad \text{If } AC = 0, \text{ GO TO } 38$$

$$9. \quad \text{If } AC > 0, \text{ GO TO } 13$$

The corresponding Mach number will be returned.

$$10. \quad \text{NU2} = X$$

Change sign of accuracy requirement.

$$11. \quad \text{ACC} = -AC$$

$$12. \quad \text{GO TO } 19$$

The Mach number after an expansion will be calculated.

First check for error conditions.

$$13. \quad \text{If } M1 < 1.0, \text{ then GO TO } 41$$

Change the angle to radians.

14.  $THETAR = (THETA)(0.0174532925)$

Define a calculational constant.

15.  $BETA1 = \sqrt{M1^2 - 1}$

Calculate the initial and final values of the P-M angle.

16.  $NU1 = NU(BETA1)$

17.  $NU2 = NU1 + THETAR$

Redefine accuracy requirement.

18.  $ACC = AC$

See if the P-M angle exceeds the maximum value.

19. If  $NU2 > NUMAX$ , then GO TO 44

Check for error conditions.

20. If  $NU2 < 0$ , GO TO 47

Make sure  $NU2$  is positive

21. If  $NU2 > 0$ , GO TO 24

The value of  $NU2$  is zero. This corresponds to a Mach number of unity.

22.  $M2 = 1.00$

23. Return

Define a counter

24.  $L = 0$

Calculate the argument of the function.

25.  $BETA2 = \sqrt{G2}$

Increment the counter.

26.  $L = L + 1$

Evaluate the function.

27.  $DNUB = DNU(BETA2)$

Recalculate another value of the argument.

28.  $BETA = BETA2 + (NU2 - NU(BETA2))/DNUB$

Determine the difference in these two values.

29.  $DIFF = ABS(BETA - BETA2)$

See if the maximum number of iterations has been exceeded.

30. If  $L > 32$ , then GO TO 36

See if a solution has been obtained.

31. If  $DIFF \leq ACX$ , then GO TO 34

Redefine argument.

32.  $BETA2 = BETA$

Repeat calculations.

33. GO TO 26

A solution has been reached. Calculate Mach number.

34.  $M2 = SQRT(B^2 + 1)$

35. Return

Error conditions have been noted. The first deals with exceeding the maximum number of iterations.

Print out the statement and return the last result.

36. PRINT 80, NU2, BETA

37. GO TO 34

The accuracy requirement is zero.

38. PRINT 101

39. PRINT 100, M1

40. RETURN 6

The initial Mach number is less than unity.

41. PRINT 102

42. PRINT 100, M1

43. RETURN 6

The P-M angle exceeds its maximum value.

44. PRINT 103

45. PRINT 99, NU2, NUMAX

46. RETURN 6

The calculated P-M angle is less than zero.

47. PRINT 104

48. PRINT 99, NU2, NUMAX

49. RETURN 6